Trade Credit Default*

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Abstract

Default on trade credit repayments is substantial, accounting for about 7% of receivable accounts in the U.S. This paper studies the role of trade-credit default in the transmission of aggregate financial shocks. We first document macro facts about trade credit and its defaults. We then develop a heterogeneous-firms quantitative model in which final-good producers purchase intermediate inputs partly on trade credit before their idiosyncratic productivity is realised. The key feature of trade credit is its low seniority. Aggregate trade-credit default is priced by input suppliers, while lenders price individual bankruptcy risk in bank credit. An insurance channel and an input-pricing channel are two novel mechanisms in the model. Steady-state effects of trade-credit default are substantial. Regarding dynamics, trade credit default amplifies a financial downturn calibrated to the Great Recession by about 80%, with amplification operating primarily through the insurance channel.

Keywords: trade credit default, delinquency and bankruptcy, entry and exit, heterogeneous firms, transmission and amplification of shocks, financial shocks

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1 Introduction

Default on trade credit payments is sizable. Jacobson and von Schedvin (2015) show that trade creditors incur significant losses from payment failures, representing a channel through which

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This paper's objective is to quantify the contribution of trade credit default to the economy's response to aggregate shocks and to identify the channels through which it operates. Using both aggregate and firm-level data for the United States, we first document key macroeconomic facts on trade credit and default, including that associated creditor losses are sizeable. We then investigate the determinants of trade credit default and its macroeconomic implications. As a preview, we develops a heterogeneous-firm model with endogenous credit risk that highlights two channels: an input purchasers' *insurance* effect and an input suppliers' *input-pricing* effect. While both shape steady-state allocations and the transmission of aggregate shocks, it is primarily through the insurance channel that trade credit default substantially amplifies a financial downturn calibrated to the Great Recession.[2]

We build a quantitative general-equilibrium heterogeneous-firms model with four types of agents: a representative input supplier producing intermediate inputs using labour; heterogeneous final-good producers that use these inputs to produce a final good; a representative household that acts as consumer, shareholder, bond holder and worker; and banks that intermediate deposits and lend to final-good firms. A fixed fraction of inputs is purchased on within-period trade credit, meaning inputs are delivered at the beginning of the period and payment is due at the end, after final-good firms observed their idiosyncratic productivity shocks. Final-good firms may also hold non-contingent bank debt or hold liquid assets. Borrowing and saving mainly reflect liquidity and precautionary motives, but are also influenced by the managerial incentive to divert liquid resources, generating an agency friction along the lines of Arellano, Bai, and Kehoe (2019). Operating final-good firms cannot issue new equity, implying non-negative dividends; dividends cannot be positive in the event of bankruptcy or trade-credit delinquency, reflecting partial creditor recovery. Trade credit debt is junior to bank debt, consistent with legislation and practice.[3] Its junior status is the fundamental reason why trade credit has any role in this model.

Liquidation by a final-good firm entails exit, possibly through bankruptcy, and implies default on trade credit. Firm's may also default on trade-credit obligations while continuing to honour their other financial obligations, a situation we refer to as delinquency. Delinquency results in a loss of

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[1] Including, for instance, the vast body of work emphasising the role of financial frictions in fluctuations since the seminal Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997b).

[2] A financial shock is an instance of driving factors considered in the rapidly growing literature on aggregate fluctuations with dynamic firms which, without being comprehensive, includes Jermann and Quadrini (2012), Khan and Thomas (2013), Bloom (2009) and, more recently, Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), Khan, Senga, and Thomas (2016), Arellano, Bai, and Kehoe (2019) and Ottonello and Winberry (2020).

[3] This is a fact well acknowledged and documented in the literature, e.g., Cuñat and Garcia-Appendini (2012) and Jacobson and von Schedvin (2015), resting under the legal protection of contractual subordination provisions contained in bank loan covenants, e.g., Bratton (2016). The junior status of trade credit is itself the subject of research, e.g., Longhofer and Santos (2003), Zhang (2019) and Garcia-Appendini and Montoriol-Garriga (2020).

future output for a random number of periods. Final-good firms face idiosyncratic productivity shocks that are realised after inputs have been purchased. Because of this timing, firms may be unable to meet all financial obligations (i.e., they become liquidity constrained). The aggregation of default on trade-credit payments determines a trade-credit loss rate that is factored into the input pricing by intermediate producers, while firm bankruptcy risk is reflected in bank lending rates.

This model contains two novel mechanisms. The first is an *insurance* channel as delinquency helps avoid the risk of costly liquidation. Its implications are two-sided. On one hand, the option of delinquency to avoid liquidation and exit supports a firm's demand for inputs. Liquidation carries costs, including the forgone continuation value of the firm and the expense on potentially idle cash inputs. Delinquency avoids these costs and thus hedges some of the risk associated with larger input purchases, making them comparatively more desirable. On the other hand, however, this element of insurance means weaker precautionary motives and lower buffer stock of liquid assets. All in all, in response to aggregate shocks, the insurance channel may initially dampen recessions by smoothing the reduction in input demand and limiting firm exit but, on the other hand, can ultimately lead to a sharper decline in input demand and deeper recessions due to reduced self-insurance and weaker productivity among surviving firms.

The second mechanism is an *input-pricing* channel. Trade-credit default—arising from both delinquency and the liquidation of final-good firms—results in losses for intermediate-input suppliers who therefore require a wedge of input prices over unit labour cost to compensate for the risk. This channel can also work in opposing directions. Following an aggregate shock, the impact rise in trade-credit default and the associated wedge would lead to a combination of higher input prices and lower wages, the former imposing higher operating costs to final-good firms, and the latter discouraging labour supply and the production of inputs. However, as weak firms exit, trade-credit default risk declines, lowering the price wedge and offsetting the initial contractionary forces, thereby potentially attenuating the overall downturn.

Parameter values are chosen so that the stationary equilibrium of the model matches a number of empirical targets based on U.S. firm-level and aggregate evidence on firms' indebtedness, operating profits, trade credit, trade-credit default, and bankruptcy. The calibrated model closely matches these data targets and also performs reasonably well regarding other non-targeted moments.

We first consider the steady state. The presence of trade-credit default reduces firms' savings and increases their debt, raises input demand, and lowers liquidation rates. These steady-state effects are quantitatively significant and translate into higher levels of net output and consumption. These outcomes are due entirely to the insurance channel. The input-pricing channel of trade credit default raises the equilibrium price of inputs and hence operates in the opposite direction, offsetting a substantial part of the effects.

To address the central question of this paper, we study the economy's dynamic response to an aggregate financial shock. We choose the shock so that the baseline model produces a GDP response as close as possible to the U.S. GDP dynamics during the Great Recession.[4] The model

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[4] Other papers that have analysed the same episode include, to name but a few, Christiano, Eichenbaum, and Trabandt (2015), Gertler and Gilchrist (2018) andArellano, Bai, and Kehoe (2019).

generates two consecutive annual declines in GDP and a cumulative output loss amounting to a large fraction of that measured in data.

We study the amplification effects of trade-credit default by comparing the response to the shock in the baseline model with that in a counterfactual model without trade-credit default. We find that trade-credit default amplifies the cumulative decline in output by about 80%, a result driven mainly by the *insurance* mechanism. On impact, the insurance element to trade credit allows more marginal firms to continue operating and weakens precautionary savings motives. As a consequence, the economy is subsequently left with a weaker pool of firms—in terms of both TFP and buffer-asset positions—which slows input hiring and the recovery in firm survival rates, thereby deepening the recession.

Turning to the *input-pricing* channel, the trade credit loss rate rises sharply on impact and, through the input-pricing mechanism described above, would initially contribute to a deeper recession in the presence of trade credit. However, the trade-credit loss rate reverts quickly and even undershoots its stationary value so that the input-pricing channel dampens—rather than amplifies—the overall cumulative decline.

We contribute to the literature at various levels. First, we address the question of the role of endogenous trade credit default in macroeconomic fluctuations. The recent, and notable, papers that study trade credit default, Boissay and Gropp (2013), Jacobson and von Schedvin (2015) and Costello (2020), are empirical micro studies. The seminal Kiyotaki and Moore (1997a)'s interest is in the transmission of defaults through credit chains at the micro firm level, an idea that Boissay (2006) extends to look at macro implications but in a static partial-equilibrium setting. Antràs and Foley (2015), in the context of international trade, study the implications of given trade credit default but do not consider its endogenous determination.[5] The emerging literature on trade credit (e.g., Altinoglu (2021) and Luo (2020)), including those with a more macro leaning (e.g., Reischer (2020) and Bocola and Bornstein (2023)) do not study default. The facts we report, and especially the measure of losses on trade receivables based on firm-level data, are relatively unknown. Some literature have also looked at cyclical patterns albeit with a different focus, e.g., Reischer (2020), Miranda-Pinto and Zhang (2020) and Boissay (2006), where only the latter have considered measures of losses to trade credit.

The second type of contribution is in the modeling. The representation of trade-credit default—and the associated insurance and input-pricing mechanisms—is unique to our framework.[6] One strand of literature on trade credit, including Altinoglu (2021), Luo (2020), and Reischer (2020), highlights the role of financial network chain linkages but in settings that are largely static and abstract from trade-credit default. As in our paper, trade credit is taken as exogenous in these papers, or specified as an exogenous function of credit spreads in Reischer (2020). In contrast with these papers, we have a fully specified intertemporal model where spreads and financial conditions are endogenously determined by equilibrium repayment risk. While network effects may represent an additional transmission channel, our input-pricing channel already captures

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[5] Other important works on international trade that study firms' financial conditions and trade finance include Manova (2013), Chor and Manova (2012), and Schmidt-Eisenlohr (2013).

[6] In Antràs and Foley (2015), exogenous default risk lowers exporters revenues affecting the availability of trade credit rather than any equilibrium prices or decisions under uncertainty.

spillovers of a similar nature to those operating through the credit chains of, e.g., Kiyotaki and Moore (1997a). Bocola and Bornstein (2023) is closer to our paper in its macroeconomic scope and the two are complementary, but they differ fundamentally in perspective. Their downstream firms need credit to operate, and trade credit's primary function is to alleviate this financing friction. In our model, in contrast, no such friction exists and trade credit's sole function is to make input purchases ex post defaultable. While in their paper reputational constraints enforce repayments, in our paper trade-credit default occurs in equilibrium. The way a financial shock affects the economy therefore differs across the two models. In their case, it involves a contraction in trade credit that worsens the input-financing friction. In contrast, in our model—given that such friction is absent—the shock transmits via a tightening of bank credit conditions. Their model abstracts from entry and exit, while firm entry and exit play an important role in our analysis.[7]

Our paper also relates to the quantitative macro literature with heterogeneous firms and bankruptcy risk, including Arellano, Bai, and Kehoe (2019), Khan, Senga, and Thomas (2016) or Ottonello and Winberry (2020). In these papers, bankruptcy risk determines a firm's access to bank credit. Our model embeds this standard mechanism of bankruptcy-risk pricing of bank debt,[8] but includes additional new channels. The insurance mechanism operates precisely through bankruptcy and exit as well as precautionary motives, and thus becomes directly relevant for this literature. Regarding the input-pricing mechanism, because trade-credit default in our model also arises from firm liquidation, bankruptcy risk interacts with aggregate spillover effects from the pricing of intermediate inputs. In this way, our model assigns a novel macroeconomic role to bankruptcies and indebtedness.

The third contribution concerns the quantitative exercise. The aggregate shock we consider relates to the financial shocks widely studied in macroeconomics with dynamic firms (e.g., Jermann and Quadrini (2012), Khan and Thomas (2013), Khan, Senga, and Thomas (2016), Mehrotra and Sergeyev (2021)) as well as in the DSGE literature we cite in Section 6 below. Our findings that trade-credit default can play an important role in the transmission of such shocks is therefore informative for that body of research. The trade credit literature has also studied financial shocks, e.g., Altinoglu (2021), Reischer (2020) and Bocola and Bornstein (2023). Compared in particular with the latter paper, we likewise find large amplification from trade credit while, in our analysis, it arises from distinct novel mechanisms associated specifically with trade-credit default.

In the remainder of the paper, Section 2 documents empirical evidence, Section 3 sets out the model, Section 4 presents the calibration and evaluates it, Sections 5 and 6 analyse, respectively, the steady state and the transmission of the shock. Section 7 concludes.

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[7] The two papers differ in other interesting modeling aspects. In Bocola and Bornstein (2023) trade credit is represented by long-term relationships and determined by a reputational constraint that enforces repayments and rules out defaults, while credit limits ensure repayment of bank credit via a static enforcement condition. That paper postulates an exogenous fraction of input purchases that needs financing, with within-period bank credit covering the shortfall when trade credit is insufficient. In contrast, in our model of anonymous decentralized one-period contracts, trade-credit default occurs in equilibrium, and bank-credit prices and limits are determined by forward looking forecasts of liquidation risk. Our trade-credit share of input purchases is exogenous, while it is intertemporal—rather than within—bank credit that may cover cash flow shortfalls.

[8] While our model shares these paper's simple representation of bankruptcy as liquidation, Corbae and D'Erasmo (2021)'s focus is on the finer elements of the actual bankruptcy code. Recent work incorporates relationship contracts; see Ferreira and Kozeniauskas (2025) for an evaluation.

2 Some motivating facts

This section presents empirical measures of trade credit and trade-credit default. Its purpose is to document features of the variables central to the analysis and to provide empirical targets for the quantitative model.

We draw on aggregate quarterly data from the Federal Reserve Board and the U.S. Bureau of Economic Analysis, as well as annual firm-level data from Compustat for the period 1980-2016. In addition to GDP and the price deflator, we use data on business sales (turnover) and employment, trade receivable accounts, bank charge-off rates and, crucially, receivables estimated doubtful. Doubtful accounts is the amount of all current account receivables estimated to be uncollectible and will be used to measure trade-credit default. To our knowledge, and aside from Boissay (2006), there is limited use of Compustat data on doubtful accounts in this context. Because our focus is on the implications of defaulted payments for input suppliers, we concentrate on evidence related to account receivables. Definitions, data sources and details of the variables constructed are in Appendix A.1 for aggregates and Appendix A.2 for firm-level data. Summary statistics are reported in Table 1.

Table 1: **Summary Statistics**

Aggregate data		*Firm-level data*	
Mean TC to GDP	0.18		
		Mean TC loss rate	0.07
Corr GDP growth with:		Corr Sales growth with:	
TC growth	0.52	TC growth	0.57
TC/GDP change	0.07	TC/Sales change	-0.16
		TC-loss-rate change	-0.43
Corr GDP HP-cycle with:		Corr Sales HP-cycle with:	
TC cycle	0.68	TC cycle	0.66
TC/GDP cycle	0.37	TC/Sales cycle	0.09
		TC-loss-rate cycle	-0.42

Note - Summary statistics for trade credit (TC) and loss rate. The mid section represents cyclical variation as the log difference for quantity variables or the simple difference for rate variables. The bottom section represents cyclical variation as the Hodrick-Prescott de-trended cyclical component of the log of quantity variables or the value of rate variables. Smoothing parameter is 1600 on quarterly aggregate data and 100 on annual firm-level data. Data sources: Federal Reserve Board, BEA and Compustat, 1980-2016. See Appendix A.1 and A.2 for definition and construction of variables.

Aggregate data - We start with aggregate data from the U.S. Financial Accounts and NIPA. We construct the time series of the ratio of trade credit receivables to GDP over the period since 1980 and find that the trade-credit-to-GDP ratio has stayed consistently around 18% throughout, with no apparent trend. This would be confirmed with the series extended further back to 1960. This ratio obviously varies at high frequency but, except for a spike around year 2000, appears to remain within a fairly narrow band.

Considering its cyclical properties, trade credit is procyclical, though less strongly so than the main macroeconomic aggregates. This is evidenced by the positive correlations of 0.52 between log changes in trade credit receivables and GDP, and 0.68 between the HP-filtered cyclical components of receivables and GDP. On the other hand, the correlation between the change in the ratio of trade credit to GDP and the log change in GDP is a very weak 0.07, while the correlation between the HP-filtered cyclical components of the trade-credit-to-GDP ratio and GDP is a higher but still modest 0.37. It suggests that the ratio of trade credit to GDP exhibits only limited cyclical variation.

Firm-level data - We turn now to firm-level Compustat data and construct variables that can be compared to the aggregates just discussed. From the individual-firm data, we have obtained yearly aggregates of trade credit receivables and sales.The cyclical behaviour of trade credit receivables can be described by the correlation of its log change with the log change in total sales, or the correlation between the HP-filtered components of receivables and sales. At 0.57 and 0.66 respectively, these correlations are comparable to the corresponding figures obtained on aggregate data, describing trade credit as procyclical. Regarding the ratio of trade credit to sales, the correlation between changes in the receivables-to-sales ratio and the log change in sales is -0.16, while the correlation between the HP-filtered cyclical components of the ratio and sales is 0.09. Both coefficients are small, with one being negative. Firm-level data supports therefore the weak cyclicality of the share of trade credit that we found on aggregate data. Existing firm-level evidence is also broadly consistent with limited cyclicality of aggregate trade-credit shares.[9]

We turn now to trade credit default, a variable unavailable on aggregate time series. Based on the Compustat firm-level variables for doubtful accounts receivable and trade credit receivables, we construct firms' ratios of doubtful accounts to receivables as a proxy of trade-credit default, or expected trade-credit loss, for suppliers. We obtain aggregate yearly observations as the average of this measure of the trade-credit loss rate across firms within each year. The average ratio of receivables doubtful over receivables, doubtful or not, gives an empirical loss rate on trade credit of 6.6% over the period. This rate is not far from the estimate in Jacobson and von Schedvin (2015) of 8% for Sweden based on direct administrative data. The time series displayed in Figure 1 contains two visible spikes, a sharp one coinciding with the early 2000's dotcom bust, and a smaller one during the 2008 financial crisis. Regarding its cyclical properties, at annual frequency, the correlation of changes in the trade-credit loss rate and sales growth has a negative coefficient of -0.43, and the correlation between the HP-filtered components of the loss rate and sales is -0.42. The loss rate thus appears to be countercyclical.[10]

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[9] Using Compustat data for the U.S., Miranda-Pinto and Zhang (2020) find that the ratio of receivables to sales fluctuates only moderately, even rising initially before declining by 10% during the Great Recession. Related evidence during financial crises in Asia and Mexico is provided by Love, Preve, and Sarria-Allende (2007) and Love and Zaidi (2010). Evidence from Europe also suggests modest cyclicality: Coricelli and Frigerio (2019) find that firms experienced increases in net trade credit during the Great Recession; McGuinness and Hogan (2016) find patterns consistent with countercyclical trade credit among SMEs.

[10] For comparison, we can consider a measure of losses associated with bank loans to firms. Based on aggregate data

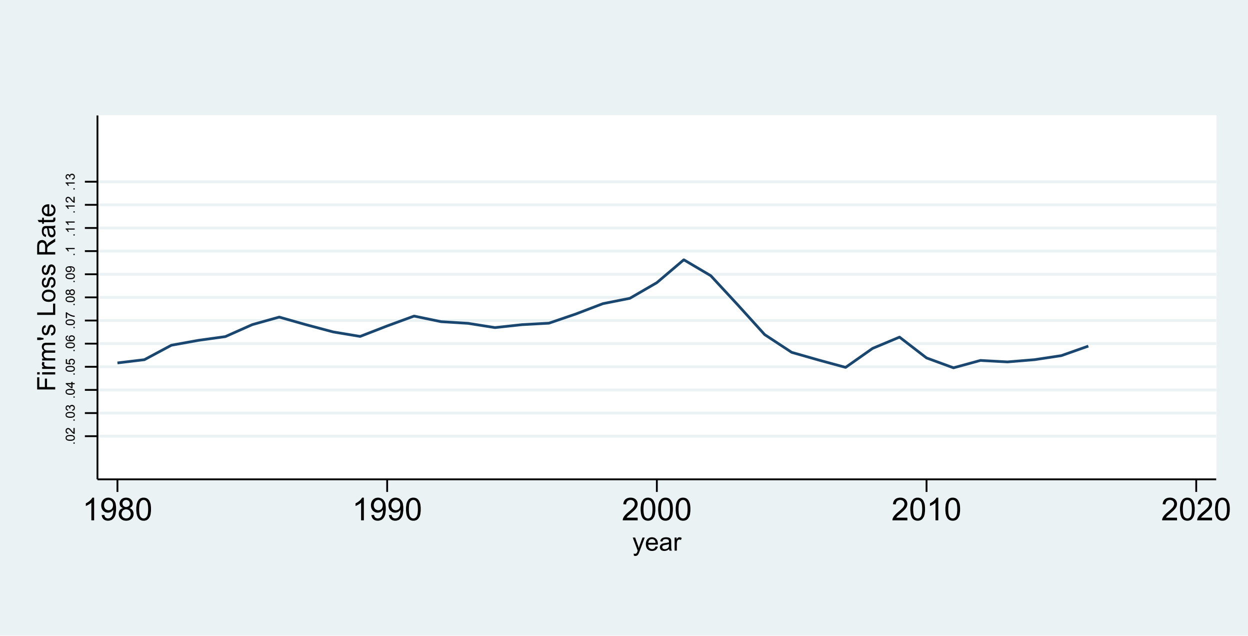

Figure 1: Trade credit default rate.

Note - Time series of trade credit default. Source: Compustat 1980-2016. Year average firm-level trade-credit loss: doubtful accounts receivable to trade credit receivables. See Appendix A.2. Authors' calculations.

3 Model

The model features four types of agents: intermediate-input producers, final-good producers, financial intermediaries, and households. All firms are competitive, and both inputs and goods are homogeneous. Final-good firms face standard constraints on equity issuance and are subject to idiosyncratic productivity shocks. They purchase a fraction of inputs on trade credit, with payment due only after shocks are realised. In the absence of commitment to repay this within-period trade credit, firms may default on suppliers and become delinquent. Final-good firms can also save or borrow through banks, with spare borrowing capacity subject to an agency problem. They may exit through liquidation and declare bankruptcy. We now introduce the notation and the key assumptions of the model; formal details are provided further below.

3.1 Assumptions

Input producers - There is a unit mass of intermediate-input producers. The intermediate input x is produced using labour n one-for-one, so that $x = n$.[11] The input price is p. Payments from retail

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from Fed described in Appendix A.1, charge-off rates on business loans are only 0.78% on average, and countercyclical, with a correlation of changes in charge-off and GDP growth of -0.25.

[11] Our model might seem isomorphic to a model where labour is the intermediate input. But that is not the case since then: (1) wage commitments would be senior to any other liabilities; (2) our modeling would impose unrealistically highly centralised wage bargaining; and (3) our data on trade credit receivables would not apply since default on wages is too rare in practice (see also Arellano, Bai, and Kehoe (2019) on this point).

customers are received with a within-period delay. A fraction τ of sales is conducted on trade credit, of which a proportion θ, representing the trade-credit loss rate, will not receive payment. The remaining fraction of sales, $1-\tau$, is on cash and receive payment with certainty.

The assumption of a constant trade-credit share is standard in the literature (see Section 1). Incidentally, the evidence suggests that this share may be only mildly cyclical (see Section 2). Given our focus on default, treating τ as constant provides a parsimonious benchmark.[12] The assumption that input suppliers pool repayment risk θ across buyers can be justified by the widespread use of standarised trade-credit contracts at the industry level, as documented by Klapper, Laeven, and Rajan (2012) to motivate a similar assumption. Such standarisation is consistent with an environment in which suppliers do not observe buyer characteristics beyond order size.[13]

The cash-flow to the representative input producer includes the costs of labour at wage w. There is free entry in the input producing sector. The trade-credit loss rate θ is forecast at entry. With free entry, expected profits are zero, that is $px - \theta\tau px - wx = 0$, given information available at the beginning of the period. The wedge of the input price over the wage rate, p/w, is thus determined by the default forecast θ.

Final-good producers - Output of a final-good firm, y, depends on aggregate productivity z, idiosyncratic productivity ϵ, and intermediate inputs x through a production function $F(\cdot)$ according to $y = ze^{\epsilon}F(x)$. Idiosyncratic productivity follows a Markov chain with transition probabilities $\psi_{\epsilon}(\epsilon'|\epsilon)$ over support $\mathcal{E}$. There is free entry. To enter, a firm pays the entry cost ξ^E, financed through equity issuance. Upon entry, the firm draws an initial productivity ϵ_{-1} from the stationary distribution $\overline{\psi}_{\epsilon}(\epsilon_{-1})$. Each period, operating the firm requires a fixed cost c_F, which includes, for example, capital replacement and the manager's wage, w_m, equal to an exogenous outside option.

At the beginning of each period, the firm chooses input demand x before the realisation of the idiosyncratic shock. After observing the shock ϵ, the firm can issue one-period debt $b > 0$ at discount price q, or save by holding cash, that is negative debt $b < 0$, at discount price Q_0. It also decides whether to default on trade-credit payments, $d^x \in \{0,1\}$, and whether to liquidate, $d^b \in \{0,1\}$. Default on trade credit ($d^x = 1$) triggers a penalty equal to a fraction $\tilde{\nu} > 0$ of output in subsequent periods, capturing disruptions to production. Let $\nu \in \{0, \tilde{\nu}\}$ denote the delinquency state. Conditional on no further default, the penalty is forgiven with probability λ each period. If the firm liquidates ($d^b = 1$), it exits; when $b > 0$, liquidation entails bankruptcy on bank debt.

Under bankruptcy or delinquency, claimants—creditor banks and trade-credit suppliers—receive the firm's residual value, if positive, with recovery r^b accruing to banks and r^x to trade-credit

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[12] The constant fraction of trade-credit sales then renders largely irrelevant the price differentiation between cash and credit inputs since they are sold as a bundle. For ease of exposition, we assume away such price differentiation.

[13] Even if the seller could infer repayment risk from order size, price differentiation based solely on the amount purchased would not stand before U.S. competition law. The Robinson-Patman Act, 15 US code §13, forbids price discrimination, and contemplates companies adjusting terms, including denial of credit, but only on reasonable standards of credit worthiness. The observed industry practice of uniform pricing suggests that non-financial firms have only limited capacity to observe and act on reasonable credit worth characteristics. Explaining the observed practice as an endogenous outcome within a firm-to-firm long-term relational contract setting—of the type in Antràs and Foley (2015) or Bocola and Bornstein (2023)—might affect the extent of the model's price pass-through mechanism and would be an interesting extension of our work.

suppliers according to their priority. Because firms must cover the operating fixed cost c_F and cash input payments $(1 - \tau)px$, the residual value of operating may be negative. In that case, liquidation implies the firm cannot cover the fixed cost and must cease operations (i.e., cease production).

The final-good firm maximises the expected discounted value of dividends, using the household's equilibrium discount factor ρ. While operating, the firm faces the constraint that it cannot issue new equity, so dividends must be non-negative.[14] As noted, however, dividends may be negative for firms that are liquidating and, at the same time, must cease current operation. Because of creditor recovery, a firm is also constrained not to pay positive dividends or retain earnings (i.e., save cash) when liquidating or becoming delinquent. Bank debt is senior to trade credit: trade credit suppliers are repaid only after bank claims are satisfied. As discussed in Section 1, this seniority structure reflects institutional features of credit markets. The preceding description means that cash input payments are senior to all other claims, while equity payouts constitute the most junior claim.

Finally, the firm faces an agency problem. Each firm employs a manager who lives for one period (i.e., the myopic case, as in Arellano, Bai, and Kehoe (2019)) and, because of lack of commitment, may have incentives to divert some of the firm's liquid resources at the end of the period. If no diversion occurs, the manager earns the wage w_m at the firm. Alternatively, the manager can appropriate available liquid funds within the firm to operate a side project, using the same production function as the firm but scaled by a factor η, and privately extract the resulting profits. The liquid resources available for diversion consist of both internal cash holdings and any unused borrowing capacity. This agency friction problem means the firm will seek to restrict its liquidity buffer to limit diversion incentives, inducing a participation constraint.

Financial intermediaries - Lenders extend one-period loans to final-good firms. They have the same information as firms, so loan contracts are contingent on firm characteristics and loan size. Competition drives expected profits on each contract to zero. In this way, debt discount prices q reflect the bankruptcy risk— implied by the firm's decision d^b and recovery r^b — over the market discount price Q. Loan prices also include an exogenous wedge or spread χ, capturing financial frictions.These lenders fund their loans by selling securities to households at discount price Q.

Intermediaries hold firms' cash deposits at discount price Q_0, and use the funds to purchase risk-free securities at the market discount price Q. There is an intermediation cost which, under free-entry competition in intermediation, drives a wedge $q_{\text{spr}} \geq 1$ between the firms cash savings and the market rates of return, so that $Q_0 = Q \times q_{\text{spr}}$.

Households - Households own the firms and the labour force. There is a representative infinitely-lived household who can borrow and lend freely at the market discount price Q. Given a subjective discount factor β and period utility $u(c, l)$, the household chooses labour supply l and consumption c.

Equilibrium - An equilibrium consists of decision rules, prices, and allocations such that the following conditions hold: Final-good firms' borrowing, saving, and default decisions maximize firm value given input and debt prices; intermediate-input prices ensure zero profits for input producers, given wages and the aggregate trade-credit loss rate; wages and the mass of final-

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[14] Here we follow Arellano, Bai, and Kehoe (2019) and Khan, Senga, and Thomas (2016).

good firms adjust to clear the labor market and satisfy free entry; loan prices satisfy the lenders' zero-profit condition, taking firms' decision rules as given; exit flows are consistent with firms' optimal policies; households choose consumption and labor supply optimally; the distribution of firm types is consistent with these decisions and prices. In a small open economy, the risk-free discount price Q is exogenous, and the economy can borrow or lend internationally, with external debt denoted by D; in a closed economy, $D = 0$.

3.2 Recursive representation

We now cast the model in recursive form. The aggregate state at the beginning of a period, S, includes the distribution of firms and aggregate financial positions:

$$S \equiv (N \times \mu, A, D),$$

where $N \times \mu$ denotes the distribution of firms over individual states (ϵ_{-1}, b, ν) at the start of the period, with μ a probability measure and N the mass of firms; A denotes household asset holdings; and D external debt. The law of motion for the aggregate state is given by $S' = H(S)$. Entry as a fraction of incumbent firms (N), can be written $m(S)$. Equilibrium prices—the input price, wage, and trade-credit loss rate—can therefore be written as functions of the aggregate state: $p(S)$, $w(S)$, and $\theta(S)$.

The final-good firm's individual state prior to the realization of idiosyncratic shocks is (ϵ_{-1}, b, ν). After the shock is realised, the state becomes (ϵ, b, ν, x), where x is the input choice made earlier in the period.

The pricing of bank debt depends on the firm's state and the aggregate state. Let $q^{ND}(b', \epsilon, \nu|S)$ denote the discount price of debt for a non-delinquent firm, and $q^{x}(b', \epsilon|S)$ for a firm choosing delinquency. In the small open economy, the world discount price Q, the firm's discount factor ρ, and the deposit price Q_0 are constant scalars.

The remainder of this section formalises the elements required to make operational the equilibrium definition in Section 3.1; additional details are provided in Appendix B.

3.2.1 Input producers

Given $\theta(S)$, $p(S)$, and $w(S)$, the zero-profit condition for intermediate-input producers (see Section 3.1) means that the input price-wage ratio satisfies

$$\frac{p(S)}{w(S)} = \frac{1}{1 - \theta(S)\tau}. \qquad (1)$$

This price wedge increases with the trade-credit loss rate θ, reflecting the pricing of default risk in the input market.

3.2.2 Final-good firms' decisions

There are two stages to the firm's problem. Let $V(\epsilon, b, \nu, x|S)$ denote the value function in the second stage, after the realisation of the productivity shock, and $W(\epsilon_{-1}, b, \nu|S)$ the value in the first stage, before the current shock is realised. In the second stage, the firm evaluates three options: repayment V^{ND}, delinquency V^{x}, or liquidation V^{b}.

Repayment - When the firm honours all obligations, borrowing/saving b' solves

$$\begin{aligned}
V^{ND}(\epsilon, b, \nu, x|S) \quad &= \quad \max_{b' \in \mathbb{R}} \Big\{ cih(\epsilon, \nu, b, x|S) + q(b', \epsilon, \nu|S)b' + \rho \mathbb{W}'(\epsilon, \nu, b'|S) \Big\} \qquad (2) \\
\text{s.t.} \quad & cih(\epsilon, \nu, b, x|S) \equiv (1-\nu) z e^{\epsilon} F(x) - c_F - p(S)x - b, \\
& q(b', \epsilon, \nu|S) \equiv \begin{cases} q^{ND}(b', \epsilon, \nu|S) & b' \geq 0 \\ Q_0 & b' < 0, \end{cases} \\
& \mathbb{W}'(\epsilon, \nu, b'|S) \equiv \mathbf{I}_{\nu>0} \Big(\lambda W(\epsilon, b', 0|S') + (1-\lambda) W(\epsilon, b', \tilde{\nu}|S') \Big) \\
& \qquad\qquad\qquad\qquad + \mathbf{I}_{\nu=0} W(\epsilon, b', 0|S') \text{ with } S' = H(S), \\
& cih(\epsilon, \nu, b, x|S) + q(b', \epsilon, \nu|S)b' \geq 0, \\
& \mathbf{I}_{b'<0} Q_0 \, (-b') + B^{*}(\epsilon, \nu) - \mathbf{I}_{b' \geq 0} q^{ND}(b', \epsilon, \nu|S)b' \leq M(\epsilon, \nu), \\
& B^{*}(\epsilon, \nu) \equiv \max_{b' \geq 0} q^{ND}(b', \epsilon, \nu|S)b', \\
& \eta(1-\nu) z e^{\epsilon} (M(\epsilon, \nu)/p(S))^{\gamma} = w_m.
\end{aligned}$$

The first three conditions define cash-in-hand cih, the price of borrowing or saving q, and the expected continuation value $\mathbb{W}'$. The fourth condition imposes non-negative dividends. The fifth is the participation constraint: the liquid resources available at the end of the period (left-hand side) cannot exceed the level $M(\epsilon, \nu)$ that would induce managerial diversion. The last two conditions define the firm's credit line $B^{*}(\epsilon, \nu)$ and the diversion threshold $M(\epsilon, \nu)$ which makes the manager indifferent between pursuing a side project (with productivity parameter η) and remaining in the firm at wage w_m. Details of the manager's problem are provided in Appendix B.1. The solution to the problem in Eq.(2) yields the policy rule $b' = g^{ND}(\epsilon, b, \nu, x|S)$ and associated dividends

$$\begin{aligned}
\pi^{ND}(\epsilon, b, \nu, x|S) = (1-\nu) z e^{\epsilon} F(x) - c_F - p(S)x - b \\
+ \begin{cases} q^{ND}(g^{ND}(\epsilon, b, \nu, x|S), \epsilon, \nu|S) g^{ND}(\epsilon, b, \nu, x|S, z) & \text{if } g^{ND}(\epsilon, b, \nu, x|S, z) \geq 0 \\ Q_0 g^{ND}(\epsilon, b, \nu, x|S, z) & \text{otherwise} \end{cases}
\end{aligned} \qquad (3)$$

Delinquency - When the firm repudiates payments for trade-credit inputs, it determines borrowing b' and the supplier's recovery r^x according to

$$\begin{aligned} V^x(\epsilon, b, \nu, x|S) &= \max_{b', r^x \geq 0} \Big\{ cih(\epsilon, \nu, b, x|S) + q^x(b', \epsilon|S)b' - r^x + \rho \mathbb{W}'(\epsilon, b'|S) \Big\} \\ \text{s.t.} \quad & cih(\epsilon, \nu, b, x|S) \equiv (1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x - b, \\ & \mathbb{W}'(\epsilon, b'|S) \equiv W(\epsilon, b', \tilde{\nu}|S') \text{ with } S' = H(S), \\ & b' \geq 0, \\ & cih(\epsilon, \nu, b, x|S) + q^x(b', \epsilon|S)b' - r^x = 0, \\ & \mathbf{I}_{b'<0} Q_0 \, (-b') + B^*(\epsilon) - \mathbf{I}_{b'>0} q^x(b', \epsilon|S)b' \leq M(\epsilon, \nu), \\ & B^*(\epsilon) \equiv \max_{b' \geq 0} q^x(b', \epsilon|S)b', \\ & \eta(1-\nu)ze^{\epsilon}(M(\epsilon, \nu)/p(S))^{\gamma} = w_m. \end{aligned} \tag{4}$$

The first two conditions define cash-in-hand cih under trade-credit default and the continuation value $\mathbb{W}'$, noting that delinquency sets the firm into the penalised state $\tilde{\nu}$. The third condition rules out cash saving under delinquency. The fourth condition imposes zero dividends: dividends must be non-negative but are also required to be non-positive under delinquency, implying that all available resources are distributed to claimants. The fifth condition is the manager's participation constraint. The final two expressions define the firms's credit line $B^*(\epsilon)$ and the diversion threshold $M(\epsilon, \nu)$. The solution yields the borrowing policy $b' = g^x(\epsilon, b, \nu, x|S)$ and the supplier's residual recovery $r^x = r^x(\epsilon, b, \nu, x|S)$.

Appendix B.2 presents the characterisation of outcomes under the repayment and delinquency options.

Liquidation - Liquidation and exit means firms in debt declare bankruptcy, but firms without debt may also liquidate if they cannot cover the fixed cost and cash input payments. There are two possible situations. If cash-in-hand

$$(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)px + \mathbf{I}_{b<0}(-b),$$

is non-negative, the firm produces and the value of the firm is zero; any residual is recovered by debt creditors as r^b.[15] If instead cash-in-hand is negative, the firm cannot cover the fixed cost and the cost of cash inputs and therefore ceases production. In this case, there is no creditor recovery, and the firm must still cover the incurred cost of cash inputs. We represent the failure to produce by the indicator $d^f(\epsilon, b, \nu, x|S) = 1$. In sum,

$$\begin{aligned} V^b(\epsilon, b, \nu, x|S) &= \begin{cases} 0 & \text{if } m(\epsilon, \nu, x|S) \geq 0 \\ -(1-\tau)p(S)x + \mathbf{I}_{b<0}(-b) & \text{otherwise} \end{cases} \\ d^f(\epsilon, b, \nu, x|S) &= \begin{cases} 0 & \text{if } m(\epsilon, \nu, x|S) \geq 0 \\ 1 & \text{otherwise} \end{cases} \\ \text{with } m(\epsilon, \nu, x|S) &\equiv (1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x + \mathbf{I}_{b<0}(-b) \end{aligned} \tag{5}$$

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[15] We are only describing situations where cash in hand is less than outstanding $b > 0$. This simplifies notational burden as there is no recovery by trade credit in this case, an implication of the assumed seniority of bank debt. This

When $b > 0$, bank-debt creditors recover the residual value $r^b(\epsilon, b, \nu, x|S) = \max\{(1-\nu)ze^{\epsilon}F(x) - c_F - (1-\tau)p(S)x, 0\}$. Dividends π^b coincide with the firm's value under liquidation,

$$\pi^b(\epsilon, b, \nu, x|S) = V^b(\epsilon, b, \nu, x|S), \tag{6}$$

and may therefore be negative when the firm fails to operate in the liquidation period.

Choice of repayment option - The optimal choice among the three options in the second stage yields policy rules $d^x(\epsilon, b, \nu, x|S)$ and $d^b(\epsilon, b, \nu, x|S)$ as the solution to

$$V(\epsilon, b, \nu, x|S) = \max\left\{V^{ND}(\epsilon, b, \nu, x|S), V^x(\epsilon, b, \nu, x|S), V^b(\epsilon, b, \nu, x|S)\right\}. \tag{7}$$

The demand for inputs - We now turn to the first stage within the period, before the realisation of the shocks. Taking V as given by Eq.(7), the optimal choice yields the policy rule $x = x(\epsilon_{-1}, b, \nu|S)$ that solves

$$W(\epsilon_{-1}, b, \nu|S) = \max_x \sum_{\epsilon} \psi_{\epsilon}(\epsilon|\epsilon_{-1})V(\epsilon, b, \nu, x|S). \tag{8}$$

3.2.3 Entry, exit and distribution

The value of a new entrant, $W^E(S)$, is given by the expected value of $W(\cdot)$ over the unconditional distribution of the initial productivity ϵ_{-1} and debt b, μ^E, reflecting the assumption of zero initial debt financing.[16] Details are in appendix B.3. The free-entry condition is

$$W^E(S) \leq \xi^E, \tag{9}$$

with strict inequality only when entry is zero, $m(S) = 0$.

The probability measure μ is defined over ex-ante firm types (ϵ_{-1}, b, ν). We define transition probabilities for existing firms as $\text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S)$, where $\mathcal{B}'$ is a set containing elements b', and for entrants—who begin with $\nu = 0$—as $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S)$. These transition probabilities are determined by firms' optimal decisions and the process governing the delinquency flag, and they jointly induce the transition function $H^{\mu}(\epsilon, \mathcal{B}', \nu' \mid S)$. The law of motion for the mass of firms N accounts for the incument firms that survive into the next period. For outcomes affected by new firms, it is convenient to define the post-entry distribution over firm types, $\hat{\mu}(\epsilon_{-1}, \mathcal{B}, \nu)$, which combines the distribution of incumbents with that of entrants. In particular, it incorporates the entry rate $m(S)$, measured relative to the incumbent mass N, together with the pre-existing distribution μ. Further details on the distribution are provided in Appendix B.4.

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focus is at no loss because if, otherwise, cash in hand exceeds $b > 0$ then the option of liquidating with some trade credit recovery will be strictly dominated by delinquency and will never materialise.

[16] Ottonello and Winberry (2020), for instance, also make this assumption. The fraction of the entry cost financed by debt could be made positive.

3.2.4 Trade-credit loss rate

Producers of intermediate inputs take as given the expected aggregate default rate on trade credit, or lost fraction of sales on trade credit, θ, conditional on the initial state S. This trade-credit loss rate results from aggregating firms' individual delinquency decisions $d^x(\epsilon, b, \nu, x|S)$ and bankruptcy decisions $d^b(\epsilon, b, \nu, x|S)$, given that input demand is determined by $x = x(\epsilon_{-1}, b, \nu|S)$. The loss rate also depends on the recovery from delinquent firms' cash in hand remaining after repayment of debts and cash inputs, which we denote by $r^x(\epsilon, b, x|S, z)$. Specifically,

$$\theta(S) = \frac{\int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1})\Big(d^x(\cdot)(\tau p(S)x(\cdot) - r^x(\cdot)) + d^b(\cdot)\tau p(S)x(\cdot)\Big)\hat{\mu}(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)}{\int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1})\tau p(S)x(\cdot)\hat{\mu}(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)} \tag{10}$$

where, for convenience, we are using the shorthand notation $r^x(\cdot) \equiv r^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^x(\cdot) \equiv d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^b(\cdot) \equiv d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, and $x(\cdot) \equiv x(\epsilon_{-1}, b, \nu|S)$, and integration is taken over the post-entry measure $\hat{\mu}$.

3.2.5 Remaining equilibrium conditions

Lenders and intermediation - Lenders use firm's decision rules and the transition probabilities of idiosyncratic shocks to infer the probability of debt default. They also take into account the expected recovery from the firm's residual value in the event of liquidation. The price of debt can therefore be written as

$$q^{ND}(b', \epsilon, \nu|S) = \chi\, Q(1 - \Lambda^{ND}(b', \epsilon, \nu|S)) \tag{11}$$

when there is no current delinquency, and

$$q^x(b', \epsilon|S) = \chi\, Q(1 - \Lambda^x(b', \epsilon|S)) \tag{12}$$

when the firm chooses delinquency, Hare, $\Lambda^{ND}(\cdot)$ and $\Lambda^x(\cdot)$ denote the corresponding forecasts of default losses, or expected default, which depend on the liquidation (bankrutpcy) rule $d^b(\cdot)$ and the associated recovery $r^b(\cdot)$, expressed relative to debt due b'. Explicit expressions are provided in Appendix B.5. In addition, loan prices are affected by the exogenous wedge χ which shifts borrowing costs independently of default risk. Regarding liquid reserves, the intermediation spread determines the discount price for firm's cash savings, given by $Q_0 = q_{\text{spr}} \times Q$.

Households - At the beginning of each period, the representative household's state is given by (a, S), where a denotes holdings of the risk-free asset.[17] The household chooses next-period assets $a'(a \mid S)$, consumption $c(a \mid S, z)$, and labour supply $l(a \mid S)$ to solve a standard intertemporal optimisation problem. The household receives aggregate dividends $\Pi(S)$, determined by firms' dividend policies in Eq.(3) and Eq.(6) (see Appendix B.6). The first-order condition for the savings decision implies $u_c(c, l)Q = \beta u_c(c(a' \mid S'), l(a' \mid S'))$, and the condition for labour supply is $u_c(c(a \mid S), l)w(S) + u_l(c(a \mid S), l) = 0$. Recall that firms discount future expected payoffs prior

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[17] Even with aggregate shocks, contingent securities play no role here and we can think of a single bond. In Arellano, Bai, and Kehoe (2019) contingent securities are used in the context of a small open economy to provide full consumption insurance.

to the realisation of future shocks, and the appropriate rate is determined by the household's discount factor, implying $\rho = Q$. Details are provided in Appendix B.7.

Aggregation and market clearing - Aggregate consistency requires that individual asset holdings a coincide with aggregate assets A, and the evolution of the aggregate portfolio A' and aggregate labour supply $L(S)$ be consistent with the household's policy functions and the distribution of firms. The evolution of external debt, D', reflects the country's current account. Details are provided in Appendix B.8.

Equilibrium - The more formal definition is in Appendix C.1. The stationary case is defined in Appendix C.2, and the computation algorithm is in Appendix C.3.

4 Quantitative model

In this section we consider the stationary equilibrium under specific functional forms and parameter values. Numerical values for the parameters will be chosen so that the model delivers realistic levels of aggregate trade credit and default, among other observable target variables.

4.1 Specification

We specify the technology of final-good firms using the concave production function

$$F(x) = x^{\gamma}, \quad \text{with} \quad \gamma \leq 1.$$

The discrete Markov chain for the log of productivity ϵ, is constructed to approximate a continuous first-order autoregressive process with persistence ρ_{ϵ} and innovations η following an iid Normal distribution with standard deviation σ_{η}, of the form

$$\epsilon' = \rho_{\epsilon}\epsilon + \eta', \text{ with } \eta' \sim N(0, \sigma_{\eta}).$$

The approximation is implemented using a Markov chain with N_{ϵ} states, following the discretisation procedure in Tauchen (1986).

We assume an additively separable period utility function of the form

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - B\frac{l^{1+\phi}}{1+\phi},$$

where B is the weight on the disutility of work, $1/\phi$ is the Frisch elasticity of labour supply, and σ is the inverse of the intertemporal elasticity of substitution.

4.2 Calibration

A model's period corresponds to one year. In the stationary equilibrium, aggregate productivity z is a constant parameter normalised to 1. The financial wedge χ is also normalised to 1. We can set the external debt and choose $D = 0$ so that the steady state is equivalent to a closed economy.

Assigned parameters - The directly assigned parameters are summarised in Table 2. The discount factor β equals the equilibrium discount price Q, and both are set to imply an annual rate of return of 4 percent. The intermediation wedge q_{spr} on liquid reserves implies a spread of approximately 2.5 percent, which is realistic, if somewhat conservative.[18] The labour supply parameter ϕ corresponds to a Frisch elasticity of 2, while intertemporal substitution σ is set to a standard value. The curvature of the final-good production function, γ, approximates the labour share (we can think of capital in the model as fixed and uniform across firms). We adopt a value commonly used in the literature (e.g., Corbae and D'Erasmo (2021); Khan and Thomas (2013); Arellano, Bai, and Kehoe (2019)), which, as we will show, also delivers the conventional labour share in Cooley, Prescott et al. (1995). The parameters governing the idiosyncratic productivity process, ρ_ϵ and σ_η, are taken from annual estimates based on Compustat data on operating income reported in Corbae and D'Erasmo (2021) which are broadly consistent with quarterly estimates in the literature (e.g., Khan and Thomas (2013); Arellano, Bai, and Kehoe (2019); Cooper and Haltiwanger (2006)). For the discrete approximation, we use a Markov chain with $N_\epsilon = 61$ states.[19]

Table 2: **Directly Assigned Parameters.**

parameter	value	observation
productivity	$z = 1$	normalisation
financial wedge	$\chi = 1$	normalisation
subjective discount rate	$\beta = 0.9615$	4% annual return
discount price	$Q = 0.9615$	4% annual return
liquid return	$q_{\text{spr}} = 1.025$	2.5% spread
risk aversion	$\sigma = 2.0$	standard
utility labour	$\phi = 0.50$	Frisch elasticity 2.0
curvature final goods	$\gamma = 0.60$	60% approx labour share
persistence	$\rho_\epsilon = 0.653$	Corbae and D'Erasmo (2021)
volatility innovation	$\sigma_\eta = 0.20$	Corbae and D'Erasmo (2021)
number of productivity states	$N_\epsilon = 61$	

Assigned parameters from direct observations and values used in the literature.

Internally calibrated parameters - The remaining seven parameters are chosen so the model matches a set of targets. These parameters are the share of intermediate-input sales conducted on trade credit, τ; the fixed cost, c_F; the manager's outside option, η;[20] the delinquency penalty, $\tilde{v}$; the probability of forgiveness, λ; the entry cost, ξ^E; and the utility weight of labour, B. Specifics of the calibration procedure are provided in Appendix C.5.

Targets in the model - We target seven moments of the model, capturing firms' indebtedness and

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[18] See, e.g., Federal Reserve time-series evidence of short-term bills.

[19] Furthermore, as explained in Appendix C.4, we will introduce type I extreme-value shocks to final-good firms' repayment choices and input hiring decisions that follow Gumbel distributions with a common dispersion parameter $\sigma_\zeta = 0.05$. This parameter's value, while helping convergence in computations (see, e.g., Mateos-Planas, McCrary, Rios-Rull, and Wicht (2022)), is of little consequence in that it will render delinquency and liquidation practically as binary outcomes.

[20] The manager's salary w_m and η cannot be identified separately. All that matters is η/w_m. To reduce notation burden,

operating income, bankruptcy, trade-credit losses, the scale of trade credit, and employment. For debt, we use the average ratio of debt to operating income across firms, conditional on operating income being positive. For each firm, debt is given by the state variable b, while operating income is defined as revenues net of variable and fixed cost, that is $(1-\nu)ze^{\epsilon}F(x) - px - c_F$. The fraction of firms with strictly positive debt b provides the second debt-related moment, while the fraction of firms with strictly positive operating income provides the third moment.

The bankruptcy rate in the model is defined as the fraction of firms that liquidate while holding positive debt. It is computed by integrating the bankruptcy policy rule $d^b(\epsilon, b, \nu, x)$ over the ex-post distribution of firms $\mu^{\text{ex-post}}(\epsilon, b, \nu, x)$—that is, after their choice of input x and realisation of productivity ϵ—for strictly positive debt, $b > 0$. Trade-credit losses are measured as the fraction of intermediate input sales on trade credit that fail to receive payment, net of recoveries in cases of default without liquidation. In the model, this corresponds to the loss rate θ defined in Eq.(10).

For the size of trade credit, we target the ratio of trade credit to GDP, defined as the value of intermediate inputs sold on trade credit relative to the value of final output. Let $\bar{x}$ denote total input demand, obtained by integrating the input demand policy function $x(\cdot)$ over the distribution of firms, and let $\bar{y}$ denote aggregate final output, given by the integral of firm output $(1-\nu)e^{\epsilon}F(x)$ over the ex-post distribution of operating firms.The target ratio of trade credit to GDP is therefore $\tau p\bar{x}/\bar{y}$. Finally, labour supply is given by the household's optimal choice l.

Empirical targets - The empirical counterparts to the targeted moments described above are constructed from aggregate and firm-level data. For debt and operating income, we use firm-level data from Compustat for the period 1980-2014; details are provided in Appendix A.3. In the data, we measure debt net of savings. This definition corresponds to the model's endogenous state, which represents the (negative of the) firm's net financial position. Since the model focuses on precautionary motives and abstracts from decisions on fixed and long-term investments, savings are measured in the data as liquid assets. For debt, we use a measure of short-term debt liabilities, which aligns closely with the scope of the model. The resulting empirical targets are an average ratio of net debt to (positive) operating income of -0.61, a fraction of firms with positive net debt of 0.35, and a fraction of firms with positive income of 0.75.

For the empirical counterpart to the trade-credit loss rate in the model, we draw on the evidence presented in Section 2. Using Compustat 1980-2016, we computed the ratio of receivables doubtful over the sum of all receivables, doubtful or not, which motivates an empirical target loss rate on trade credit of 7 percent. For the size of trade credit, we use aggregate time series from the Federal Reserve and BEA on receivables and GDP to construct the ratio of trade credit to GDP, which informs our target ratio of 0.18.

For the bankruptcy rate target, we draw on estimates from the related literature. Corbae and D'Erasmo (2021) report a rate close to 1 percent based on Compustat data and about 2 percent using a distance-to-default measure. Ottonello and Winberry (2020) report a default rate of around 3 percent based on business survey data. We therefore target an intermediate value of 2 percent. Finally, the target for employment is normalised to unity.

Calibration results - Parameter values are obtained by minimising the sum of squared deviations

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we fix $w_m = 0.25$, and take η as the parameter to calibrate. We can always renormalise so that, as required, $w_m \leq c_F$.

between model-implied moments and empirical targets.[21] The calibrated parameters and targeted variables are summarised in Table 3.

Table 3: **Calibrated Parameters**

parameter	calibrated value	target variables
fixed cost c_F	0.760	fraction in bankruptcy
side project η	0.078	debt to operating inc
penalty size $\tilde{\nu}$	0.153	fraction in debt
penalty forgiveness λ	0.332	fraction with positive operating inc
entry cost ξ^E	9.028	trade-credit loss rate
input sales on trade credit τ	0.290	trade credit to GDP
weight of work in utility B	2.458	labour supply

Values of the seven internally calibrated parameters (left panel) and list of the targeted variables (right panel).

This calibration implies that approximately 29 percent of input sales are on trade credit. The delinquency penalty parameters imply a future loss of about 15 percent of output over an average period of 3 years. The total fixed operating costs incurred by final-good firms amount to roughly 21 percent of the value of aggregate final output—above, but not far from, the replacement cost of capital of 16 percent implied by the capital-to-output ratio of 2.3 and depreciation rate of 6.9 percent in, for instance, Khan, Senga, and Thomas (2016).

Table 4 reports results under the calibrated parameter values. The first column contains the empirical targeted moments and the second column the values implied by the model. The model proves a close match to the data, particularly considering the parsimonious model and that the complex interactions complicate identification.[22]

Table 4: **Moments in model and data**

Moments	Data	Model
fraction in bankruptcy	0.02	0.02
(net) debt to operating inc	-0.61	-0.61
fraction in debt	0.35	0.35
fraction with positive operating inc	0.75	0.74
trade-credit loss rate	0.07	0.07
trade credit to GDP	0.18	0.17

Values of targeted variables in the data and the calibrated model. Trade credit to GDP from Federal Reserve and BEA 1980-2016. Debt, operating income and trade-credit loss rate from Compustat 1980-2016. See Appendices A.1, A.2 and A.3. Author's calculations. Bankruptcy is based on Corbae and D'Erasmo (2021) and Ottonello and Winberry (2020).

For a sense of external validity, about 47 percent of firms in the model have strictly positive

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[21] The algorithm is based on the Software BOBYQA, authored by M. J. D. Powell, to minimize sum of squares with bound constraints by combing trust region method and Levenberg-Marquardt method.

[22] Labour supply, not shown, matches its normalisation target exactly.

dividends, compared with approximately 51 percent in Compustat data. The model's endogenous exit rate is about 2.72 percent, which lies in the range between the exit rate of 1.20 percent in Corbae and D'Erasmo (2021) and the 5.2 percent in Khan, Senga, and Thomas (2016).[23] The model implies a labour share of GDP of 0.58, broadly in line with global and U.S. measures by the 2010's (see Karabarbounis and Neiman (2014) and Ilo (2015)), supporting the initial choice of γ. Mateos-Planas, Seccia, and Yavuzoglu (2025) show that cross-sectional implications of this model compare well with the data.

5 Trade credit default in the steady state

In this model, the absence of trade credit default is equivalent to the absence of trade credit. To illustrate the channels through which trade credit default operates, we therefore compare the benchmark steady state with trade credit to the steady state of a counterfactual economy without trade credit, holding all other parameters fixed.

The first column of Table 5 reports the percentage changes in steady-state variables implied by the introduction of trade credit (and associated default), relative to the steady state without trade credit. The direction of these changes is consistent with the incentives and equilibrium price effects discussed in Section 1, namely the *insurance* channel and the *input-pricing* channel.

Table 5: **Effect of trade credit default in steady state (percent change)**

	general equilibrium	partial equilibrium
Firm entry and exit:		
entry rate m	-33.2	-41.5
fraction liquidating	-32.4	-40.4
Debt:		
mean debt	61.4	65.2
fraction in debt	48.6	70.0
Firm-level outcomes:		
average input per firm	6.1	13.4
average output per firm	2.7	7.0
GE prices and aggregates:		
input price p	3.5	0.0
wage rate w	1.4	0.0
number of firms N	-5.6	0.0
net output/consumption	1.0	15.5

Note: Percentage differences between steady state with trade credit and steady state without trade credit. The first column is in general equilibrium. The second column is in partial equilibrium, holding fixed the prices and number of firms of the steady state with trade credit. Average output and input are over the $N(1+m)$ firms that hire inputs at the beginning of a period. Net output/consumption is final output minus entry costs and fixed costs incurred.

The main implications of trade credit are lower asset holdings—equivalently higher debt—for

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[23] The latter paper targets a total 10 percent exit rate, but with 4.8 percent consisting of exogenous departures.

and lower liquidation and exit rates. The *insurance channel* associated with trade-credit default weakens precautionary saving motives, reducing firms' demand for buffer assets (and increasing their reliance on debt) and raising their demand for inputs. Trade-credit default also directly lowers the probability of liquidation by allowing firms to avoid costly exit, thereby reducing exit rates and, hence, entry rates in the steady state. Quantitatively, these effects are sizeable. At the aggregate level, the presence of trade credit increases net output and consumption, reflecting higher firm-level input hiring and lower resources devoted to entry costs, despite a smaller mass of firms.

Turning to prices, the input price increases by about 3.5 percent with trade-credit default. Free-entry bidding is stronger under trade credit because it raises the expected value of entry. The wage also increases, though more modestly (by 1.4 percent), consistent with the intermediate supplier's zero-profit condition, reflecting the raised wedge of input prices over wages required to compensate for trade-credit default risk. This is the *input-pricing* channel.

To assess the importance of these general-equilibrium adjustments, we consider the steady state without trade credit in partial equilibrium, holding prices (the input price and wage) and number of firms fixed at their benchmark levels under trade credit.[24] We then examine the changes that occur when introducing trade credit. The second column of Table 5 reports these responses. Relative to the first column, the increases in the entry rate, aggregate net output, firms' input demand and output, and debt are substantially larger. This comparison highlights the role of general-equilibrium adjustments associated with trade-credit default: the increase in the input price dampens both the expansion in firms' input demand and the decline in exit (and entry). These equilibrium adjustments, together with the reduction in the number of firms, significantly moderate the increase in output associated with the introduction of trade credit.

6 Trade credit default in a recession

This section studies the role of trade credit default in the transmission of an aggregate shock. Since the model does not incorporate aggregate uncertainty, we consider unanticipated, deterministic changes in the path of a model parameter.[25] This requires extending the model to include a time varying exogenous aggregate state.[26] In solving the model, we must account for the possibility of strictly zero entry during contractionary periods following a negative shock.[27] Because we consider a small open economy along the transition path, the price of bonds remains constant and the market clearing condition in consumption may require external borrowing/saving during the transition,

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[24] In this exercise we thus omit market clearing and zero-profit conditions; m will still be endogenous to satisfy a constant mass of firms N in steady state.

[25] In the spirit of the so called MIT shocks. See, e.g., Boppart, Krusell, and Mitman (2018), Guerrieri and Lorenzoni (2017).

[26] When the shock follows an AR(1) process, this amounts to adding the corresponding state variable to the aggregate state vector S. Alternatively, one can index endogenous value and policy functions, as well as exogenous shocks, by time. Our computation follows this time-indexing approach.

[27] In contrast with the equilibrium logic when $m >> 0$, the free-entry condition (9) becomes a strict inequality when $m = 0$, so the wage will now have to fall to clear the labour market, and it is the price of inputs that will then adjust to meet the pricing condition (1) rather than meeting the entrants' zero-profit condition.

as in Arellano, Bai, and Kehoe (2019).[28] Consumption and external debt therefore respond to reflect the wealth effect of the shock. Implementation details are provided in Appendix E.

Several model parameters could serve as sources of aggregate shocks. For concreteness, we choose the 2007-2009 Great Recession in the U.S. as the reference event. Motivated by the central role of financial frictions during this period, we model the shock as a reduction in the path of the lending wedge, χ in Eq.(11) and (12), capturing a decline in the exogenous component of the discount price of firms' debt, or equivalently, an increase in the lending spread. This specification is closely related to the risk-premium or spread shocks in Christiano, Eichenbaum, and Trabandt (2015), Christiano, Motto, and Rostagno (2014), or Gilchrist and Zakrajsek (2012).

6.1 Size and duration of the Great Recession

We take 2007:IV and 2009:II as the start and end dates of the Great Recession, consistent with the chronology of the NBER Dating Committee and with related studies, including Arellano, Bai, and Kehoe (2019). Figure 2 reports the cyclical component of GDP (the output gap), based on trends fitted over the pre-recession period 1985:I-2007:III —HP-filtered trend in one case and linear trend in the other case— extrapolated over the recession period.[29] The vertical line marks the pre-recession quarter 2007:III. The cumulative output loss over the two-year long recession corresponds to the widening of the gap between 2007:III and 2009:II shown in Figure 2, amounting to -8.1 percent under HP de-trending, and -8.8 percent under linear de-trending.

6.2 The financial shock

We specify the shock so that, although unanticipated, it becomes known to all firms at the beginning of the impact period (period 0), before idiosyncratic shocks are realised, and will thus cause no losses or gains for input producers. To be specific, along the transition, the debt pricing function in Eq.(11) becomes

$$q_t^{ND}(b_{t+1}, \epsilon_t, \nu_t) = \chi_{t+1} Q(1 - \Lambda_{t+1}(b_{t+1}, \epsilon_t, \nu_t))$$

(and similarly for (12)), where the wedge χ_{t+1} is observed at time t.[30] The initial shock at time zero consists therefore of a fall in χ_1 below the steady state value $\chi_0 = 1$. We assume the shock follows an AR(1) adjustment process so that it is fully described by the initial innovation and its persistence. Specifically, given $\chi_1 < \chi_0 = 1$, for $t = 2, 3, \ldots$ the log of χ_t evolves according to $\log \chi_t = \rho_\chi \log \chi_{t-1}$, where $\rho_\chi < 1$ is the persistence of the shock.

We choose the parameters of the financial shock to generate a contraction that, as closely as possible, matches the movements of U.S. GDP during the Great Recession. Exploring alternative values of

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[28] Alternatively or in addition to considering an open economy, other restrictions like constant number of firms, are typically made in the literature studying dynamics, for instance Ottonello and Winberry (2020), Arellano, Bai, and Kehoe (2019) and Bordalo, Gennaioli, Shleifer, and Terry (2021).

[29] Other papers similarly look at sample starting in 1985, e.g., Arellano, Bai, and Kehoe (2019) and Christiano, Eichenbaum, and Trabandt (2015).

[30] It is dated $t + 1$ only for computational convenience as the pricing of debt is determined within the backward loop of the algorithm.

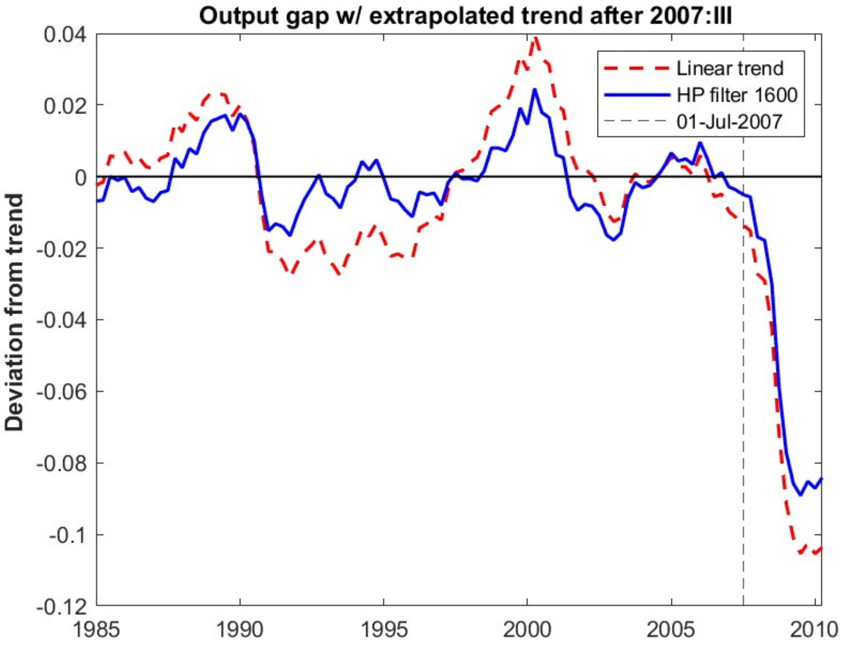

Figure 2: Output gap 1985:I-2010:II. Trends fitted to 1985:1-2007:III data, and extrapolated afterwards. Solid line for HP trend; dashed line for linear trend. HP trend extrapolated via quadratic polynomial fitted to data 2005:II-2007:III.

χ_1 and ρ_χ indicates that the shock must be quite severe to generate two consecutive declines in output rather than a single drop on impact. Even so, the model can account for at most two-thirds to three-quarters of the observed cumulative output loss of approximately 8 percent. We set $\chi_1 = 0.02$ and $\rho_\chi = 0.10$, which yield a cumulative GDP decline of 5.4 percent over two consecutive periods before recovery begins. Although smaller than in the data, the implied recession remains substantial. The relatively low persistence is also plausible. The initial shock produces a spike in lending spreads that effectively shuts down credit on impact but—in contrast to cases with higher persistence—spreads subsequently fall sharply, to an average of about 25 percent in the periods corresponding to 2009–2010, reflecting the shock's rapid dissipation. It is also consistent with the view that the financial disturbance was largely transitory.[31] We assess robustness to these assumptions below.

6.3 The recession

Figure 3 displays the paths of variables following the shock. The decline in output over the two periods (top panel) is associated with a collapse in entry and a sharp increase in the liquidation rate on impact (middle panel). The number of firms declines as a consequence, reaching a cumulative loss of about 6 per cent at the trough of the recession (bottom panel).

The response of output reflects the joint movements of the number of firms (through entry and

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[31] E.g., Gertler and Gilchrist (2018).

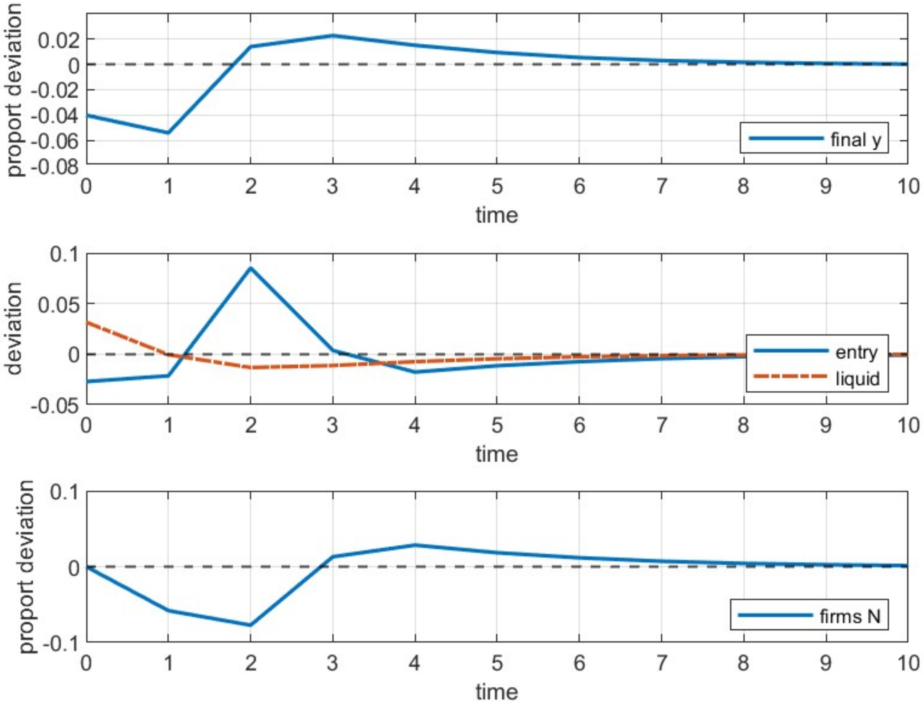

Figure 3: Baseline economy—response to the financial shock. Quantities: output (top), entry and exit/liquidation (middle), stock of firms (bottom). The top and bottom panels report proportional deviations from the initial steady state; the middle panel reports deviations from steady state.

exit), input use, and their productivity. We decompose final output y into its components: the stock of firms at the beginning of period, N; the gross entry factor $1 + m$, with m the entry rate; average input x purchased per firm; firm survival rate (or 1 minus exit rate); and the average productivity of firms surviving to the end of the period, denoted tfp′, as follows:[32]

$$y \approx \underbrace{N \times (1 + \text{entry})}_{\text{number of firms}} \times \underbrace{(\text{aver input})^{0.6} \times \text{survival}}_{\approx\text{endogenous output per firm}} \times \underbrace{\text{aver survivor's tfp}'}_{\approx\text{residual output}}. \tag{13}$$

Table 6 reports percentage changes in output and its components. On impact, the decline in output is driven by lower entry, reduced survival (higher exit), and lower input use per firm. Entry collapses to zero and liquidation rises sharply, generating a positive selection effect that raises the productivity of surviving firms (tfp′).

In the second period, the further decline in output is accounted for almost entirely by the reduced number of firms remaining and the fact that entry, though positive, stays below trend. Partially offsetting these forces, survival rates and input hiring per firm recover, reflecting in part the earlier

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[32] This is an approximation since it ignores higher moments of the cross sectional distribution of inputs and survival. The residual tfp′ should capture them.

Table 6: **Changes in components of GDP since before shock**

period	y	N	1+entry	aver input	survival	tfp′
1st	-4.01	+0.00	-2.64	-0.82	-3.24	+2.36
2nd	-5.41	-5.80	-2.09	+5.17	+0.06	-0.68

Percentage changes relative to initial steady state. The "tfp′" column obtains as the residual. The contribution of input hiring to changes in GDP can be approximated as the entry under "aver input" multiplied by 0.6, the production function's γ.

positive selection effect.

In sum, the cumulative output loss is driven by the decline in the number of operating firms, resulting from the surge in exit on impact and weak level of entry throughout the recession. It is therefore an *extensive-margin* recession; on the intensive margin, input hiring by the typical surviving firm increases, partly offsetting the cumulative contraction in aggregate output.

These cumulative changes reflect both the direct effect of the shock and equilibrium adjustments. The smaller but more productive pool of firms operating at the start of the second period results from the collapse in entry and surge in exit on impact. Both responses follow directly from tighter credit conditions, which render entry unprofitable while forcing marginal firms into liquidation, thereby strengthening the productivity composition of surviving firms.

The reduced entry rate in the second period reflects equilibrium in the labour market. Tighter credit lowers the value of entry, putting downward pressure on input prices and wages. In equilibrium, lower wages are associated with reduced labour supply and, consistent with market clearing, reduced firm entry. Changes in input prices and wages are reported in the two leftmost columns of Table 7. On the intensive margin, however, incumbent firms increase input demand in the second period despite tight financial conditions, reflecting both the positive selection generated by earlier exit and the lower input prices.

Table 7: **Changes in prices and trade-credit default since before the shock**

period	p	w	delinquency	tc loss (θ)	input wedge (p/w)
1st	-0.68	-1.72	+5.86	+3.53	+1.04
2nd	-2.23	-1.49	-0.69	-2.56	-0.74

Changes relative to initial steady state. The columns "delinquency" and "tc loss" contain level changes in percentage points. Columns "p", "w" and "input wedge" represent percentage changes.

Turning to trade-credit default, the three right-most columns of Table 7 report changes in the proportion of delinquencies, the trade-credit loss rate θ, and the implied wedge of the input price over the wage $p/w = 1/(1 - \theta\tau)$ (see Eq.(1)). On impact, delinquencies rise by about 6 percentage points and the trade-credit loss rate increases by 3.5 points (see Figure 7 below for the full path of the loss rate θ). These losses are priced through a 1.04 percent increase in the wedge. In the second period, delinquencies—and thus trade-credit losses—fall below steady-state levels, reflecting both reduced debt exposure and the higher productivity of surviving firms following the initial selection effect.

These movements in trade-credit default must contribute to the cumulative changes in output components reported in Table 6. Because delinquency carries a future penalty, its surge on impact must have contributed to the reduced productivity tfp' at the end of the second period (corresponding to a 15 percent penalty applied to the additional 6 percent of delinquent firms). The swings in the average demand for inputs per firm—declining on impact and rising in the second period—also partly reflect corresponding swings in the price wedge.

6.4 Amplification role of trade credit default

Having characterised the recession, we now evaluate the role of trade-credit default. How does allowing for trade-credit default affect the response of aggregate output to the financial shock, and through which mechanisms? We focus on its contribution to the cumulative decline in output. To this end, we compare the baseline economy to a counterfactual without trade-credit default, which in this model is equivalent to eliminating trade credit altogether.

We re-calibrate this no-trade-credit economy. In the absence of trade credit, several parameters become irrelevant, leaving the manager's outside option η, the fixed costs c_F, the entry cost ξ^E, and the labour supply weight B to be determined. We target the same moments as in the baseline—debt-to-operating-income, the fraction of firms in debt, and normalized labor supply—but replace the bankruptcy rate with the steady-state entry rate. Our robustness analysis below indicates this choice is conservative for assessing amplification.

Table 8 reports the re-calibrated parameters and the targeted moments. Compared to the baseline (Tables 3 and 4), the counterfactual features a higher manager's outside option and a lower fixed cost, which together sustain the targeted levels of debt and entry. The lower fixed cost compensates for the absence of the insurance provided by trade-credit default in the baseline model.

Table 8: **Calibration of zero trade-credit default counterfactual**

Parameters		Moments	
fixed cost c_F	0.58	debt to operating income	-0.61
entry cost ξ^E	12.82	entry rate	2.72%
weight of work in utility B	2.13	fraction in debt	0.36
side project η	0.09	labor supply	1.00

The table reports parameter values and targeted moments for the economy without trade-credit default. Targets include the entry rate (in place of the bankruptcy rate). The left panel reports calibrated parameters, and the right panel reports the corresponding model-implied moments.

The steady states of the counterfactual and baseline economies are closely comparable, which facilitates interpretation. Both feature a debt-to-income ratio of 61 percent, an entry rate of 2.72 percent, and a liquidation rate of 2.64 percent. We consider alternative counterfactual specifications in robustness analysis below.

We next examine the dynamic response of the no–trade-credit counterfactual economy to the financial shock. Figure 4 plots the paths of the main aggregates for the counterfactual (dashed) alongside the baseline paths from Figure 3 (solid). The output responses differ substantially over the transition. During the recession period (periods 0 and 1), the baseline economy exhibits a visibly larger cumulative output loss, implying that trade credit default amplifies the recession. This amplification occurs despite the sharper rise in exit—and the associated faster decline in the number of firms—in the no–trade-credit counterfactual.

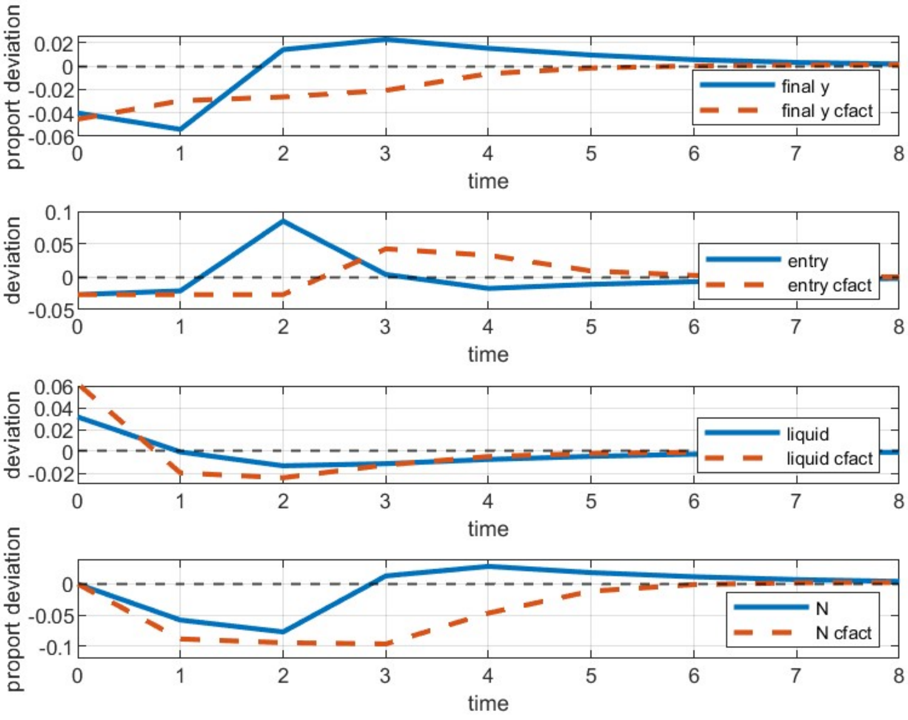

Figure 4: Baseline and counterfactual—response to the financial shock. Baseline (solid) and no-trade-credit counterfactual (dashed). Variables: output (top), entry (second), exit/liquidation (third), and number of firms (bottom). The top and bottom panels report proportional deviations from the initial steady state; the second and third panels report deviations from steady state.

To quantify these differences, we compare cumulative percentage changes in GDP and its components in the baseline and counterfactual economies. Table 9 (second row) reports the differential responses—defined as baseline minus counterfactual—along with the implied amplification measure. Trade credit increases the cumulative output loss by 2.4 percentage points, corresponding to an amplification of 82 percent measured as the extra loss in the baseline relative to the loss in the counterfactual. The larger decline under trade credit reflects a markedly slower recovery in both input hiring and firm survival (by -8.8 and -2.0 percentage points, respectively), despite milder reductions in the number of firms and in entry (3.1 and 0.6 percentage points, respectively).

We now turn to the interpretation. The weaker cumulative recovery of input demand and firm survival in the baseline reflects a pool of firms with lower productivity and weaker asset positions at the start of the second period. The -3.0 percentage point difference in productivity (equivalent to -56.5 percent) reported in Table 9 (first row) arises because firm survival declines by 3.1 percentage points less on impact, thereby limiting the extent of positive selection in the presence of trade credit

Table 9: **Effect of trade credit on changes in components of GDP**

period	amplification	y	N	1+entry	aver input	survival	tfp′
1st	+11.6%	+0.5	+0.0	+0.0	+0.7	+3.1	-3.0
2nd	-82.4%	-2.4	+3.1	+0.6	-8.8	-2.0	+1.2

Column "amplification" reports the additional extra percentage change in y in the baseline relative to the percentage change in the counterfactual. All other entries report the difference in percentage changes (from initial steady state) between baseline and counterfactual. A negative sign represents an amplifying effect of trade credit default on the corresponding variable's decrease, while a positive sign indicates a dampening effect .

(see Figure 6 below for the full tfp path). The response of asset positions also differs across the two economies. In the baseline, deleveraging and the accumulation of cash buffers proceed more slowly. As shown in Figure 5, the decline in (net) debt obligations is smaller, implying a reduction by the second period that is about 15 percent lower than in the counterfactual.

Both responses on impact—the smaller decline in firm survival and the smaller increase in cash savings—stem from the insurance provided by trade-credit default. By relaxing repayment obligations on impact, the availability of trade-credit default reduces exit and precautionary saving, but in doing so weakens selection and slows balance-sheet repair. As a result of it, firms enter the second period with weaker balance sheets and lower productivity, which dampens the recovery in input demand and firm survival. The financial shock therefore produces a deeper recession in the baseline economy through the *insurance* channel first described in Section 1.[33]

In Appendix D, we examine alternative specifications of the counterfactual and the shock process and find that the amplification result is robust. Table 11, in that appendix, reports the corresponding outcomes in a format that helpfully highlights the role of productivity and debt positions as proxies for aggregate states across specifications, including the benchmark case analysed here.

We now turn to the role of the input-pricing channel. Alongside the differences in GDP components discussed above, the two economies differ in the adjustment of other equilibrium variables, including delinquency, trade-credit losses, and prices. Changes in these variables for the baseline were reported in Table 7 while Table 10 now reports price responses for both the baseline and the counterfactual.[34] In the baseline, although wages decline slightly less on impact, they fall substantially more in the second period relative to the counterfactual (indeed, they increase marginally in the latter). This pattern is consistent with—through labour supply—the amplification of the cumulative output loss in the presence of trade credit. Regarding input prices, their cumulative decline in the baseline (-2.2 percent) exceeds that of wages (-1.5 percent), reflecting the fall in trade-credit loss rate θ (see Table 7), which is captured by the reduction in the price wedge p/w (-0.74 percent). This decline in wedge supports the recovery in input demand per firm in the baseline relative to the counterfactual, where the wedge stays constant by construction. Therefore, the

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[33] In the first period, as shown (see Table 9), the presence of trade credit implies more moderate declines in both survival (by 3.1 percentage points, as already discussed) and input per firm (by 0.7 percentage points), resulting in a modest attenuation of the output contraction on impact (11.6 percent). This initial dampening is largely the counterpart of the substantial cumulative amplification.

[34] Note the equivalent changes in p and w within the counterfactual since there is no changes in the pricing wedge in that case.

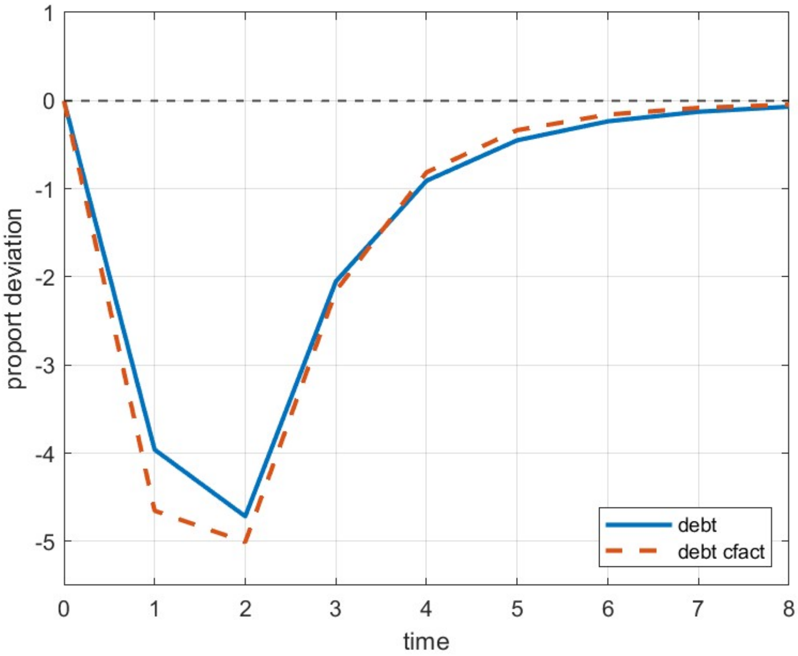

Figure 5: Net debt in baseline and counterfactual—response to the financial shock. Baseline (solid) and no-trade-credit counterfactual (dashed). Variables: net debt per firm. Proportional deviations from initial steady state.

input-pricing channel of trade-credit default described in Section 1 dampens, rather than amplifies, the cumulative output loss during the recession.

Table 10: **Effect of trade-credit default on price changes**

	period	p	w	p/w
Baseline (trade credit)				
	1st	-0.7	-1.7	+1.04
	2nd	-2.2	-1.5	-0.74
Counterfactual (no trade credit)				
	1st	-2.2	-2.2	0.00
	2nd	+0.5	+0.5	0.00

Changes in input prices, p, and wages, w, and the input-price wedge p/w relative to initial steady state. Percentage changes. The top panel refers to the baseline economy, and the bottom panel to the counterfactual.

6.5 Post-recession adjustment

So far, we have focused on differences between the baseline and the counterfactual during the recession period. However Figure 4 shows that the two economies follow markedly different paths thereafter. Following the recession, the trade-credit baseline recovers more rapidly: output even overshoots its steady state shortly after the trough, whereas output in the counterfactual remains subdued.[35]

The faster recovery in the baseline is associated with higher entry, lower survival, and a stronger improvement in the legacy tfp component relative to the counterfactual. The dynamics of entry and survival after the second period (period 1) were shown earlier in Figure 4, while Figure 6 reports now the evolution of tfp as an aggregate-state proxy. In fact, the combination of lower survival and higher entry in the baseline are behind the selection effect that generates this positive differential in productivity. At the same time, higher entry supports a faster rebuilding of the stock of firms: the number of firms increases in the baseline, while it continues to decline for some time in the counterfactual. The earlier recovery of entry in the baseline—equivalently, the delayed recovery in the counterfactual—emerges as the primary driver of the divergence in adjustment paths.

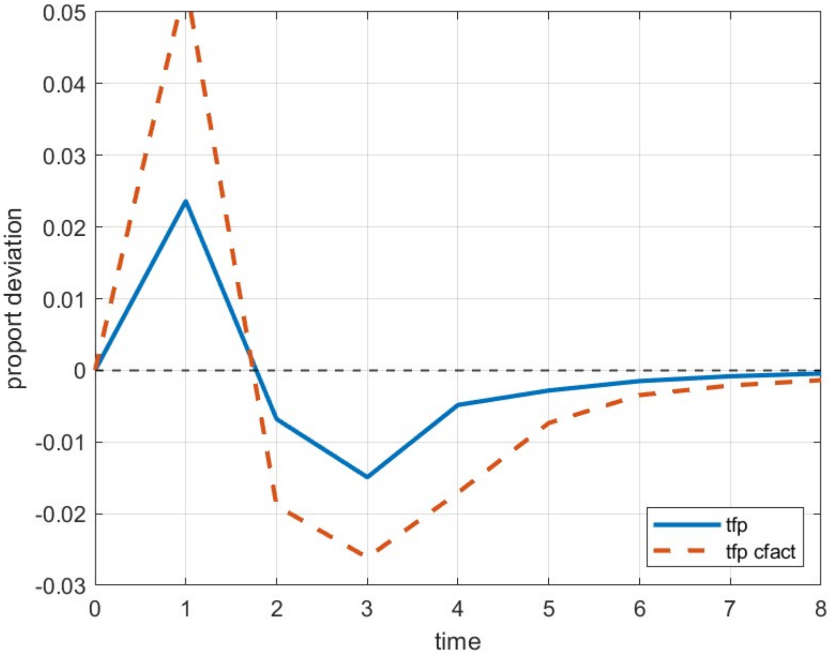

Figure 6: tfp in baseline and counterfactual—response to the financial shock. Baseline (solid) and no-trade-credit counterfactual (dashed). Variables: tfp productivity residual of surviving firms at the end of the previous period. Proportional deviations from initial steady state.

What explains the contrasting behaviour of entry after the recession? In the baseline, although credit conditions remain tights and continue to depress firm values after the recession, entry resumes relatively quickly because input costs decline. Lower input prices allow new firms to break even despite the still-elevated financial shock. The decline in input prices arises from a reduction in delinquency risk, which compresses the price-wage spread following the recession. Figure 7 illustrates the joint evolution of the input price (p) and the trade-credit loss rate (θ).

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[35] Overshooting mainly reflects elevated entry and lowered delinquency and hence a stock of firms above steady-state level.

In contrast, in the no-trade-credit counterfactual inputs become more expensive. Absent the trade-credit risk-spread mitigation channel, input prices must track the recovering wages. As a result, firm values remain too low to sustain entry for several additional periods. Entry resumes only once both the input price and the financial shock have returned sufficiently close to their steady-state levels, by the 4th period (by period 3 in Figure 7).

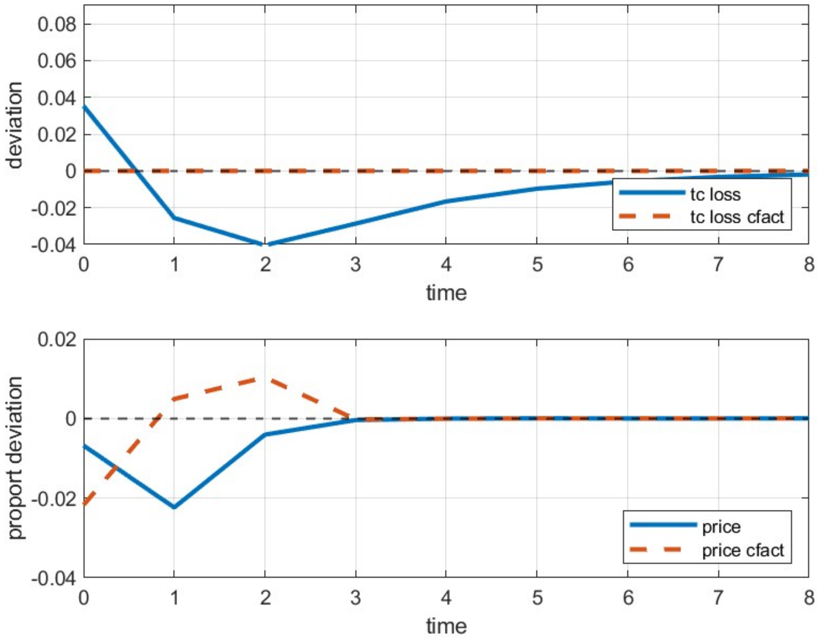

Figure 7: Trade-credit loss rate and input price in baseline and counterfactual—response to the financial shock. Baseline (solid) and no-trade-credit counterfactual (dashed). Variables: trade-credit loss rate θ (top), and input price p (bottom). Units: deviation and proportional deviation, respectively, from initial steady state.

7 Concluding remarks

This paper asks whether trade-credit default is quantitatively important for understanding macroeconomic fluctuations. We show that it can be. We study a large financial recession driven by a surge in credit spreads. The option to default as insurance against liquidation results in a weaker pool of surviving firms—in terms of both productivity and balance sheets—and a deeper recession after the initial impact. This occurs despite the countervailing changes associated with the pricing of trade-credit risk. Our results arise from the interplay between these two forces, with the insurance channel dominating.

The mechanisms that trade-credit default introduces should be relevant for analysing other shocks and policies, and for deriving empirical implications and testing them. An important direction for further work is to incorporate the determination of trade credit through the interaction between upstream and downstream firms. Having made here a case for the macroeconomic significance of trade-credit default, much work remains.

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A Appendix: Data

A.1 Aggregate data for evidence about trade credit

We describe the aggregate data used for Section 2.

Series retrieved from FRED Federal Reserve Economic Data, Economic Research Division, Federal Reserve Bank of St. Louis. Link: `https://fred.stlouisfed.org/series/`

Period and frequency: 1980-2016, quarterly.

Sources:

- Trade receivables: Board of Governors of the Federal Reserve System (US), Z.1 Financial accounts.
- GDP and deflator: U.S. Bureau of Economic Analysis (BEA)
- Charge-off rates: Board of Governors of the Federal Reserve System (US).

Variables:

- Trade receivables accounts:

  TRABSNNCB =Nonfinancial Corporate Business; Trade Receivables; Asset, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted

  TRABSNNB = Nonfinancial Noncorporate Business; Trade Receivables; Asset, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted

- Implicit GDP deflator:

  GDPDEF = Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted

- Real GDP:

  GDPC1 = Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate

- Charge-off rates:

  CORBLACBS = Charge-Off Rate on Business Loans, All Commercial Banks, Percent, Quarterly, Seasonally Adjusted

Notes: The series for trade credit have been seasonally adjusted. GDP has been inflated to nominal to construct ratios of trade credit. The trade credit variables have been deflated to construct levels of time series.

A.2 Firm-level data for evidence about trade credit

We describe the firm-level data used for Section 2.

Source: Compustat. Accessed trough Wharton Research Data Service (WRDS) Compustat - Capital IQ, North America, Fundamentals Annual.

Period 1980-2016, annual frequency.

Exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000.

Variables:

- Trade credit: measured as *Accounts receivables - trade* (`rectr`);
- Trade-credit default: measured as *Receivable estimated doubtful* (`recd`). Although the estimated doubtful accounts variable in Compustat includes both trade and non-trade accounts, trade accounts represent the vast majority, 90%, of all receivables (`rect`), and only about 2.5% of firms in the sample have any receivable accounts that include non-trade items.
- Revenues or output: measured as *Sales* (`sale`).
- Employment: measured as *Employment* (`emp`)

Notes for aggregate analysis in Section 2: Trade credit loss rate must be constructed as `recd` over the sum `rectr+recd`. Construct time series as aggregates by year of receivables, sales, average trade credit loss level, and average trade credit loss rate, and then the implied ratios of trade credit to sales. For measuring time series properties of level variables, deflate aggregate receivables, sales, and trade credit loss, using the same GDP deflator as in sec A.1 above. Construct cyclical variation as log differences of level variables, and the difference for the loss rate.

A.3 Debt and operating income for calibration

In the calibration Section 4.2, for debt and operating income we use firm-level data from Compustat for the period 1980-2014. Accessed trough Wharton Research Data Service (WRDS) Compustat - Capital IQ, North America, Fundamentals Annual.

The raw data is cleaned in a way similar to Corbae and D'Erasmo (2021).We have borrowed some of their Stata code. We drop observations with 0 or missing assets, sales and property, plant and equipment, and exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000. Observations are deleted if they do not have a positive book value of assets or if gross capital stock or sales are zero, negative, or missing.

Variables used for calibration as measured in Compustat:

- Operating income: measured as *Income before interest* (code `ebitda`).

- Short-term debt: measured as *Debt in current liabilities* (code `dlc`).
- Long-term debt: measured as *Long-term debt* (code `dltt`).
- Liquid savings: measured as *Cash and short term investments* (code `che`), consisting of 'Cash Special Deposits, Working Funds and Temporary Cash Investments'.

B Appendix: Details of model

B.1 The problem of the manager

At night, the manager can use available funds $M > 0$ on a side project. They can purchase inputs x so $px = M$ for a payoff $f_m(\epsilon, x) = \eta(1-\nu)e^{\epsilon}F(x)$, with same specification as for the firm $F(x) = x^{\gamma}$. Maximisation over x gives the profits the manager can extract from the side project: $\eta(1-\nu)e^{\epsilon}(M/p)^{\gamma}$. The outside value of the manager is a given exogenous w_m. Enforcement means the available funds M must satisfy $\eta(1-\nu)e^{\epsilon}(M/p)^{\gamma} \leq w_m$. This determines the implicit expression for the indifference threshold $M(\epsilon, \nu)$ in (2) and (4).

B.2 Final-good firms characterisation

The following analysis characterises the outcomes from the block of the model in Section 3.2.2.

Consider ND firms first, where the notation is as in (2). There are two cases.

- The first is when $\text{cih}(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) \leq 0$. In this case, $b' \leq 0$ and solves the savings problem $\max_{b' \in [b_L, 0]} \left\{\text{cih}(\epsilon, \nu, b, x) + Q_0 b' + \rho \mathbb{W}'(\epsilon, \nu, b')\right\}$, with the bound on the level of savings $(-b_L)$ being the minimum of the cash in hand limit and the limit ensuring enforcement:
$$(-b_L) = \min\left\{\frac{\text{cih}(\epsilon, \nu, b, x)}{Q_0}, \frac{-(B^*(\epsilon, \nu) - M(\epsilon, \nu))}{Q_0}\right\}.$$
- The second case is when, otherwise,
  - $\text{cih}(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) > 0$,
  - or $\text{cih}(\epsilon, \nu, b, x) < 0$.

  Here the solution has $b' > 0$ which is determined as follows:

  1. If $\text{cih} < 0$ and $-\text{cih} > B^*$: The option ND is not feasible so $V^{ND} = -\infty$.
  2. If, otherwise, $\text{cih} \geq 0$ (and hence $B^* - M > 0$) or $\text{cih} < 0$ and $B^* - M > -\text{cih}$, then b' solves for the root of
  $$b' q^{ND}(b', \epsilon, \nu) - (B^* - M) = 0,$$
  implying possible positive dividends.
  3. Otherwise (i.e, if $\text{cih} < 0$ and $B^* - M < -\text{cih}$), zero dividends and b' solves:
  $$b' q^{ND}(b', \epsilon, \nu) - (-\text{cih}(\epsilon, \nu, b, x)) = 0.$$

Consider now the delinquent Nx firms, with the notation in (4). We distinguish two cases:

- The first is when $\text{cih}(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) \leq 0$. In this case, $b' = 0$ and recovery $r^x(\epsilon, \nu, b, x) = \text{cih}(\epsilon, \nu, b, x)$.
- The second case is when, otherwise,
  - $\text{cih}(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) > 0$,
  - or $\text{cih}(\epsilon, \nu, b, x) < 0$.

  Here the solution has $b' > 0$ determined as follows:

  1. If $\text{cih} < 0$ and $-\text{cih} > B^*$: option Nx is non feasible and $V^{Dx} = -\infty$
  2. If, otherwise, $\text{cih} \geq 0$ (and hence $B^* - M > 0$) or $\text{cih} < 0$ and $B^* - M > -\text{cih}$, then b' solves
  $$b'q^x(b', \epsilon, \nu) - (B^* - M) = 0,$$
  with recovery $r^x = (B^* - M) - (-\text{cih})$.
  3. Otherwise (i.e, $\text{cih} < 0$ and $B^* - M < -\text{cih}$), b' solves
  $$b'q^x(b', \epsilon, \nu) - (-\text{cih}(\epsilon, \nu, b, x)) = 0,$$
  and yields zero recovery.

The discussion above assumes continuous debt price functions. Nonetheless, in this model the price of debt functions and therefore the resources raised via borrowing, may be discontinuous in the level of debt chosen. The reason is that at the level of debt where the firm may become delinquent with some positive probability at the end of the period, the marginal cost of hiring inputs drops and the firm's chosen amount of x may jump. One consequence is that the firm's choice of debt b' may raise excess resources and this residual would have to be apportioned to creditors accordingly.Our computational extension, explained in Appendix C.4, will dispense with the existence of such discontinuities.

B.3 Entry

The value of a new entrant $W^E(S)$ is the expectation of $W(\cdot)$ over the unconditional distribution on the starting ϵ_{-1} and b, μ^E:

$$W^E(S) = \int W(\epsilon_{-1}, b, \nu = 0|S)\mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{14}$$

Here the probability distribution of entrants is

$$\mu^E(\epsilon_{-1}, b) = \begin{cases} \overline{\psi}_\epsilon(\epsilon_{-1}) & \text{for } b = 0 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

reflecting the assumption of zero initial debt financing.[36] The free-entry condition is

$$W^E(S) \leq \xi^E,$$

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[36] Ottonello and Winberry (2020), for instance, also make this assumption. The fraction of the entry cost financed

with strict inequality only when there is zero entry, $m(S) = 0$.

B.4 Distribution

The probability measure μ is defined over the ex-ante firm types (ϵ_{-1}, b, ν). It evolves according to $N' \times \mu' = H^{\mu}(S)$ where, as defined earlier, $S = (N \times \mu, A)$. We define the transition probabilities for existing firms $\text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S)$ and $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S)$. These transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag as shown in (20) and (21) below.

The motion for the mass of existing firms N counts in the mass of current firms surviving into next period, thereby

$$N' = N \times \int \sum_{\epsilon, \nu'} \text{Prob}(\epsilon, \mathbb{R}, \nu'; \epsilon_{-1}, b, \nu \mid S) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ + Nm(S) \int \sum_{\epsilon, \nu'} \text{Prob}^E(\epsilon, \mathbb{R}, \nu'; \epsilon_{-1}, b \mid S) \mathrm{d}\mu^E(\epsilon_{-1}, b) \quad (16)$$

The transition function is

$$H^{\mu}(\epsilon, \mathcal{B}', \nu' \mid S) \equiv N \times \int \text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu, \mid S) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ + Nm(S) \int \text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S) \mathrm{d}\mu^E(\epsilon_{-1}, b), \quad (17)$$

where $\mu^E(\epsilon_{-1}, b)$ is the probability distribution of productivity and debt for new entrants. Now, since the distribution evolves as $N' \times \mu' = H^{\mu}(S)$, the probability measure follows

$$\mu' = H^{\mu}(S)/N'. \quad (18)$$

For calculating outcomes affected by new firms, it will be convenient to define the post-entry probability distribution over firm's types as $\hat{\mu}$. It accounts for the proportion $m(S)$ of new firms entering the market relative to the mass of firms N, as well as the firms in the probability measure μ already existing at the start of the period. Therefore $(N + Nm(S))\hat{\mu}$ is the total scaled up measure of firms. Given $m(S)$, N and μ and μ^E, the post-entry probability measure obtains as

$$\hat{\mu}(\epsilon_{-1}, b, \nu) = \frac{\mu(\epsilon_{-1}, b, \nu) \times N + \mu^E(\epsilon_{-1}, b)\mathcal{I}_{\nu=0} \times Nm(S)}{N + Nm(S)}. \quad (19)$$

More formally, μ belongs in the set of probability measures over a measurable space consisting of the set of elements of the individual final-good firm's state, and the product of the corresponding

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by debt could be made positive. In this case, the debt issued to cover the fraction of the entry cost $\alpha^E \xi^E$ is given by $b = b^E : q^E(b^E) = \alpha^E \xi^E$. Since the productivity type is unknown before entry, competitive lenders price this debt by pooling across types according to the function $q^E(b')$ such that $q^E(b) = \sum_{\epsilon_{-1}} \overline{\psi}_{\epsilon}(\epsilon_{-1}) q^{ND}(b, \epsilon_{-1}, \nu = 0)$. Here we have decided to keep $\alpha^E = 0$ for simplicity.

Borel algebras.

The transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag. For existing firms

$$\text{Prob}(\epsilon,\mathcal{B}',\nu';\epsilon_{-1},b,\nu\mid S)=\begin{cases}\psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad \nu'=0,\ \ \nu=0\\ \lambda\psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad \nu'=0,\ \ \nu=\tilde{\nu}\\ (1-\lambda)\psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad \nu'=\tilde{\nu},\ \ \nu=\tilde{\nu}\\ \psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{x}(\epsilon,b,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=1\\ & \quad \nu'=\tilde{\nu}\\ 0 & \quad\ \ \text{otherwise}\end{cases}\tag{20}$$

For new entrants, for whom $\nu=0$,

$$\text{Prob}^E(\epsilon,\mathcal{B}',\nu';\epsilon_{-1},b\mid S)=\begin{cases}\psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{ND}(\epsilon,b,0,x(\epsilon_{-1},b,0\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,0,x(\epsilon_{-1},b,0\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,0,x(\epsilon_{-1},b,0\mid S)\mid S)=0\\ & \quad \nu'=0\\ \psi_\epsilon(\epsilon\mid\epsilon_{-1}) & \text{if}\quad g^{x}(\epsilon,b,x(\epsilon_{-1},b,0\mid S)\mid S)\in\mathcal{B}'\\ & \quad d^b(\epsilon,b,0,x(\epsilon_{-1},b,0\mid S)\mid S)=0\\ & \quad d^x(\epsilon,b,0,x(\epsilon_{-1},b,0\mid S)\mid S)=1\\ & \quad \nu'=\tilde{\nu}\\ 0 & \quad\ \ \text{otherwise}\end{cases}\tag{21}$$

B.5 Lenders

In the equations for the price of debt, (11) for no-delinquency and (12) for delinquency, $S'=H(S)$, and $\Lambda^{ND}(\cdot)$ and $\Lambda^x(\cdot)$ denote the corresponding forecasts of default losses or expected default, which depend on the default rules $d^b(\cdot)$ and the recovery $r^b(\cdot)$ expressed as a rate over debt due b'. Expressions for Λ^{ND} And Λ^x follow. For a firm not defaulting, let the lender's expected recovery rate

$$rec^b(\epsilon',b',\nu'\mid S')\equiv\frac{r^b(\epsilon',b',\nu',x(\epsilon,b',\nu'|S')\mid S')}{b'}.$$

Therefore the expected default $\Lambda^{ND}(\cdot)$ in Eq.(11) can be written

$$\begin{aligned}
&\Lambda^{ND}(b', \epsilon, \nu | S) \equiv \\
&\qquad \mathcal{I}_{\nu>0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ (1-\lambda) d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) \\
&\qquad\qquad + \lambda d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\} \\
&\qquad\qquad\quad + \mathcal{I}_{\nu=0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\}
\end{aligned}$$

where $S' = H(S)$. The expected default for a firm incurring delinquency in (12) is

$$\Lambda^x(b', \epsilon | S) = \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) \Big\}$$

where $S' = H(S)$. Regarding liquid reserves, to repeat, the spread in intermediation determines the discount price for cash savings:

$$Q_0 = q_{\text{spr}} Q \tag{22}$$

B.6 Aggregate dividends

The aggregate dividend received by the household/shareholder can be represented by the expression shown in (23) for $\Pi(S)$.

Given the firms' dividend policies from (3) and (6), the aggregate dividend is

$$\begin{aligned}
\Pi(S) = \sum_{\epsilon} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \Bigg[\int \mathcal{I}_{d^b(\ldots)=0, d^x(\ldots)=0} \pi^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \\
+ \int \mathcal{I}_{d^b(\ldots)=1} \pi^d(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \Bigg] (N + m(S)N) \\
- \xi^E m(S) N
\end{aligned} \tag{23}$$

where we are using the shorthand notation $d^x(\cdot) \equiv d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$ and $d^b(\cdot) \equiv d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$.

B.7 Households

At the beginning of a period, the state for the representative consumer is (a, S), where $S = (N \times \mu, A, D)$, and a is the individual's risk-free asset.[37] The savings decision $a'(a \mid S)$, consumption $c(a \mid S, z)$, and labour supply $l(a \mid S)$ solve

$$U(a \mid S) = \max_{\{a', l\}} \left\{ u(c, l) + \beta U(a' \mid S') \right\} \tag{24}$$

---

[37] Even with aggregate shocks, contingent securities play no role here and we can think of a single bond. In Arellano, Bai, and Kehoe (2019) contingent securities are used in the context of a small open economy to provide full consumption insurance.

subject to

$$c + Qa' = w(S)l + a + \Pi(S),$$

where the components of S' obey $S' = H(S)$. As standard, the first-order condition for the savings decision implies $u_c(c, l)Q = \beta u_c(c(a' \mid S'), l(a' \mid S'))$, and for labour supply $u_c(c(a \mid S), l)w(S) + u_l(c(a \mid S), l) = 0$.

The firms' dividend policies from (3) and (6) determine the aggregate dividend received by the household/shareholder $\Pi(S)$. Details are in appendix B.6.

Recall firms discount future values expected before the realisation of future shocks. The appropriate rate is given by the stochastic discount factor based on a risk-free portfolio. From the consumption first-order condition, this means

$$\rho = Q \tag{25}$$

B.8 Market clearing

Aggregate consistency requires individual assets coincide with the aggregate:

$$a = A. \tag{26}$$

Given the household's policy functions $a'(a \mid S, z)$ from (24), the transition function for the aggregate portfolio A' is

$$H^A(S) = a'(A \mid S). \tag{27}$$

Similarly, aggregate labour supply and consumption are given by

$$L(S) = l(A \mid S) \text{ and } C(S) = c(A \mid S). \tag{28}$$

Equilibrium requires clearing in the market for labour, final output and assets. By Walras' Law, we only need to consider the first two. Clearing in the labour market means[38]

$$L(S) = N \times \int x(\epsilon_{-1}, b, \nu, \mid S) \mathrm{d}\mu(\epsilon_{-1}, b, \nu) + Nm(S) \int x(\epsilon_{-1}, b, \nu = 0 \mid S) \mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{29}$$

For final goods, the condition is

$$\begin{aligned} C(S) + \xi^E Nm(S) + D - Q\ H^D(S) = (N + Nm(S)) \\ \times \sum_{\epsilon} \left[\int \psi_\epsilon(\epsilon \mid \epsilon_{-1})((1 - \nu)z\epsilon F(x(\epsilon_{-1}, b, \nu|S)) - c_F)(1 - d^b(\cdot)d^f(\cdot))\mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \right] \end{aligned} \tag{30}$$

where $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ and $d^b(\cdot) \equiv d^d(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$. The two terms on the right correspond to existing and new entrants, respectively. The possibility of firm failure is

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[38] We could also write it in terms of $\hat{\mu}$, that is $l(S) = (N + Nm) \int x(.)\mathrm{d}\hat{\mu}(.)$, but the present renders more clearly the role of entry $m(S)$ in market clearing.

captured by the failure indicator $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ which is 1 when the firm declares bankruptcy and cannot cover the fixed cost and payments for cash inputs.

C Appendix: Equilibrium

C.1 Definition of equilibrium

An equilibrium consists of the functions:

- For final-good firms: policy rules $\{g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, \pi^{ND}, \pi^b, x\}$, and value functions $\{V^{ND}, V^b, V^x, W\}$.
- Loan price functions: q^{ND}, q^x.
- Input price function p.
- Wage function w.
- Aggregate trade-credit default function θ.
- Aggregate dividends Π.
- Policy functions for households: a', l, c.
- Transition function for firms' distribution H^μ.
- Transition function for household assets H^A.
- Transition function for external debt H^D.
- Aggregate labour supply L.
- The probability distribution of entrants μ^E (and b^E and q^E).
- The measure μ and mass N of firms.
- Post-entry measure $\hat{\mu}$.
- Rate of entry m.
- Value of entry W^E.

and, for the given risk-free discount price Q, the scalars

- Discount rate for firms and lenders ρ.
- Cash discount price Q_0.

They must satisfy the following conditions:

1. Final-good firms: Given q^{ND}, q^x, ρ, p, and H, final-good firms's outcomes g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, π^{ND}, π^b, and x, solve (2), (3), (4), (6), (5), (7), (8).
2. Lenders zero profit: Given H, Q, and d^b, r^b, and x, debt prices q^{ND} and q^x satisfy equations (11) and (12).
3. Free-entry intermediation of liquid reserves: Given Q, Q_0 satisfies (22).
4. Distribution transition: Given g^{ND}, g^x, d^x, d^b and x, the transition probabilities Prob and Prob^E are determined by (20), (21). Given those and m, N, μ, μ^E, then H^μ and $N'\mu'$ follow (17), (16), and (18).
5. Post-entry distribution: Given m, N, μ, μ^E, then $\hat{\mu}$ is given by (19).

6. Distribution of entrants: Given q^{ND}, (15) determines $\mu^E(\epsilon_{-1}, b)$, and b^E q^E.
7. Labour market clearing: The functions m, μ, μ^E, N, L, and x satisfy (29).
8. Trade-credit loss and failure: Given d^x, r^x, x, d^b, d^f, p and $\hat{\mu}$, then θ is given by (10).
9. Input-producers: The functions p, w and θ satisfy (1).
10. Free entry: Given W and μ^E, the function W^E in (14) and p satisfy (9).
11. Discount: Given Q, ρ is determined by (25).
12. Aggregate dividends: Given d^b, d^x, x, π^{ND} and π^b, m, N and $\hat{\mu}$, then Π is given by (23)
13. Consumer: Given H, Q, w and Π, the household's $a'(\cdot)$, l, c solve the problem in (24).
14. Clearing in final goods. Given C, D, m, x, $\hat{\mu}$ and N, and d^b and d^f, eq. (30) holds.
15. Aggregate consistency: a', H^A, L and C satisfy (26), (27) and (28).

By Walras' law, we omit the condition of clearing in the market for securities.

C.2 Stationary equilibrium

A stationary equilibrium is one where the aggregate state S is constant over time. Therefore, the endogenous equilibrium functions are constant functions, and prices and discount rates are scalars p, w, Q and ρ, and so will the quantities m, N, l, c, and θ. Note that, with positive liquidation, in this stationary equilibrium it must be the case that entry is strictly positive so $m > 0$.

C.3 Algorithm for the stationary equilibrium

In a stationary equilibrium, the Euler equation associated with (24) requires $\beta = Q$ and then, by (25), we can pin down the equilibrium discount as

$$\rho = Q = \beta.$$

The labour supply optimality condition associated with (24) reduces to $u_c(c, l)w(S) + u_l(c, l) = 0$. This condition implies, via (28), aggregate labour supply L as a function of c and w which here we denote $l^S(c, w)$.

For a predetermined D, a stationary equilibrium can be constructed through the following steps:

1. Guess price p.
2. Final-good firms and lenders: $p \longrightarrow q, x, g, \pi, r, d, V, W$
    (a) Firms Eq.(2) to (8): $p, q \longrightarrow x, g, \pi, r, d, V, W$.
    (b) Lenders Eq.(11) and (12): $d, r, x \longrightarrow q$.
3. Distribution entrants: By (15), 3(i) $q \longrightarrow q^E \longrightarrow b^E$, and 3(ii) $b^E \longrightarrow \mu^E$.
4. Final goods free entry: $W, \mu^E \longrightarrow W^E$ by Eq.(14).
5. Update p via Eq.(9). Back to 2.
6. Distribution: $d, g, x, \mu^E \longrightarrow \mu, m$
    (a) Guess μ.
    (b) Find m to match exit by Eq.(16).

   (c) Update μ by transition function Eq.(17), (20), (21), and (18). Back to 6b.

7. Post-entry distribution: $\mu, m, \mu^E \rightarrow \hat{\mu}$ by Eq.(19).
8. Trade credit loss: $\hat{\mu}, d, x, r, p \rightarrow \theta$ by Eq.(10).
9. Input pricing: $p, \theta, \eta \rightarrow w$ by Eq.(1).
10. Market clearing:
    (a) Guess N.
    (b) Consumption: $D, N, m, x, d, \hat{\mu} \rightarrow c$ by Eq.(30).
    (c) Labour supply: $c, w, \rightarrow L$ from (24) as given by $l^S(.,.)$ above via (28).
    (d) Labour clearing: $\mu^E, \mu, m, x, L \rightarrow N$ by Eq.(29). Back to (b).

Note the first 5 steps fully determine p iteratively. The remaining steps are direct, except for iterations in determining the distribution and market clearing.

C.4 Computation with extreme-value shocks

The main block in the characterisation of an equilibrium is the joint determination of final-good firms choices and lenders loan pricing, for a given price of the intermediate input p. Firms take as given the loan pricing functions in (11) and (12). Lenders take as given the firm's default and delinquency policy rules resulting from (2) through to (8). We define all value functions and policy functions on grids for the states ϵ, b, ν, and x.

Repayment options. The liquidation, delinquency or repayment outcome is a discrete choice variable and we extend the model with extreme-value shocks affecting these choices. In the decision problem of the firm among the three options in (7), we introduce shocks ζ^{ND}, ζ^x and ζ^b, associated with the decisions of repaying, delinquency, and liquidating, respectively. They are collected in a vector ζ. The decision problem is subject to an extra exogenous state ζ and is transformed into

$$\tilde{V}(\epsilon, b, \nu, x, \zeta) = \max\left\{V^{ND}(\epsilon, b, \nu, x) + \zeta^{ND}, V^x(\epsilon, b, \nu, x) + \zeta^x, V^b(\epsilon, b, \nu, x) + \zeta^b\right\}, \tag{31}$$

which gives decision rules $\tilde{d}^x(\epsilon, b, \nu, x, \zeta)$ and $\tilde{d}^b(\epsilon, b, \nu, x, \zeta)$. (I omit the aggregate states S for notational simplicity here.) Assume that these shocks follows a Gumbel, or Type-I extreme value, distribution with location and scale parameters μ_ζ and σ_ζ, that is $\zeta^j \sim G(\mu_\zeta, \sigma_\zeta)$, with cdf

$$F(\zeta^j) = \exp\left(-e^{-\frac{\zeta^j - \mu_\zeta}{\sigma_\zeta}}\right),$$

$$\text{mean}(\zeta^j) = \mu_\zeta + \sigma_\zeta \gamma, \;\; \text{with } \gamma = 0.5772, \;\; \text{var}(\zeta^j) = \sigma_\zeta^2 \pi^2/6, \;\; \text{mode}(\zeta^j) = \mu_\zeta.$$

Given this specification, the solution to this problem can be characterised in terms of a distribution of ex-ante probabilities among the different options, which are functions of the "fundamental" values V^j's. Specifically:

$$
\begin{aligned}
d^x(\epsilon, b, \nu, x) &= \frac{e^{V^x(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}} \\
d^b(\epsilon, b, \nu, x) &= \frac{e^{V^b(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}}
\end{aligned}
$$

On the other hand, the expected value

$$V(\epsilon, b, \nu, x) \equiv \mathbb{E}\tilde{V}(\epsilon, b, \nu, x, \zeta) = \mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log \sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}.$$

We normalise by a choice of μ_ζ so that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = 0$. It is known that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = \mu_\zeta + \sigma_\zeta\mu_\zeta + \sigma_\zeta \log J$ where $J = 3$ is the number of discrete options. Therefore our normalisation implies $\mu_\zeta = -\sigma_\zeta\mu_\zeta - \sigma_\zeta \log J$. Finally, to deal with issues of computer arithmetics in the evaluation of the exponential function, we perform the calculations under some convenient but innocuous transformations.

Demand for inputs. For a given discrete state, the decision about the quantity of input $x(\epsilon_{-1}, b, \nu)$ solving (8) can be computed on the discrete grid for x. In examples, we have found x to show discontinuities and unwarranted patterns. Although extreme-value shocks in repayment discrete choices help resolve some of these issues, we have also introduced shocks affecting the choice of x_i on the grid $X = \{x_1, ..., x_{N_x}\}$, given by ζ_i following a Gumbel distribution, which we stack in a vector ζ. The original problem in (8) is reformulated as

$$\tilde{W}(\epsilon_{-1}, b, \nu, \zeta) = \max_i \{RHS^x(x_i \mid \epsilon_{-1}, b, \nu) + \zeta_i\}_{i=1}^{N_x},$$

where we define

$$RHS^x(x_i \mid \epsilon_{-1}, b, \nu) \equiv \sum_\epsilon \psi_\epsilon(\epsilon|\epsilon_{-1}) V(\epsilon, b, \nu, x_i).$$

The solution can be described as an ex-ante probability distribution over the choices given by

$$\text{prob}^x(x \mid \epsilon_{-1}, b, \nu) = \frac{e^{RHS^x(x|\epsilon_{-1},b,\nu)/\sigma_\zeta}}{\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta}}.$$

The maximised value

$$W(\epsilon_{-1}, b, \nu) = E[\tilde{W}(\epsilon_{-1}, b, \nu, \zeta)] = \mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log\left[\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta}\right].$$

Normalising so the expected max of the shocks is zero means $\mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log N_x = 0$.

C.5 Internal calibration procedure

Since a number of targeted moments will depend directly on the price of inputs p, it is efficient to control p in order to meet those targets, and then choose deep parameters to be consistent with the chosen p.[39]

The steps of the procedure, in outline, are as follows: (i) Set p, and the five deep parameters τ, η, $\tilde{\nu}$, λ, and c_F; (ii) Solve for the firms-lenders equilibrium outcomes Eq.(2) to (8), (11) and (12); (iii) Find entrants' distribution and the value of entry in Eq.(15) and (14); (iv) Solve for the distribution of firms as in Eq.(16) to (19); (v) Calculate trade-credit default rate in Eq.(10); (vi) Calculate target moments (debts, defaults, etc) and check against data; (vii) Update p and parameters τ, η, $\tilde{\nu}$, λ and c_F; Back to point (ii) and repeat until best match to data; (viii) Back out ξ^E via free entry condition Eq.(9); (ix) Back out w via input pricing Eq.(1), the number of firms N by labour market clearing Eq.(29), consumption via clearing in final-good Eq.(30) and, finally, the parameter B to match the target for employment via the household's optimality condition solving Eq.(24).

D Appendix: Summary and Robustness

As reported in Section 6.4, trade-credit default substantially amplifies the recession, increasing the cumulative decline in output by about 82%. This amplification reflects the more muted responses of firm liquidations and savings (or deleveraging) on impact. This result is obtained under a particular parameterisation of the shock and type of counterfactual. We next consider alternative specifications.

Table 11 reports cumulative changes over the recession in the baseline economy (with trade credit) relative to those in the counterfactual (without trade credit). Each row corresponds to a different specification of the shock and/or the counterfactual experiment.

The first row reproduces the benchmark case discussed above. The first column reports that output declines by an additional 82% in the baseline relative to the counterfactual.[40] The next four columns report the relative changes in the components of GDP, while the last two columns show changes in variables that proxy aggregate states—namely tfp and debt. [41] The larger decline in output in the trade-credit baseline is associated with slower adjustment in firms' input hiring, survival rate, and the tfp state, as well as faster change in firms' debt positions (i.e., slower deleveraging). These patterns arise despite faster adjustment in the number of incumbent firms and in entry. The first row of Table 11 therefore summarises the mechanisms behind Table 9 discussed in Section 6.4. We now turn to the alternative specifications reported in the remaining rows of Table 11.

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[39] In this way, we avoid having to find p as the solution to the zero-profit condition, an implicit non-linear equation, for each trial of ξ^E.

[40] To see what this means, the fall in output of -5.41 in the baseline (shown in Table 6, second row, column y) is faster than in the counterfactual by the difference -2.4% (shown in second row, column y of Table 9) which represents a 82% of the fall in the counterfactual.

[41] The label tfp in Table 11 stands for the measured tfp residual at the end of the preceding period. It is an indicator of the productivity profile of the legacy firms entering into the current period, which will obviously influence, but not fully determine, the average profile of the firms surviving to the end of the current period tfp′.

Table 11: **Amplification results under alternative specifications**

	GDP	**Components of GDP**				**Aggreg states**	
	y	N	1+entry	aver input	survival	tfp	debt
(1i) Main	-82.4	+34.4	+20.9	-63.0	-97.0	-56.3	+14.9
(1ii) Liq targ	-132.1	+31.4	+9.6	-62.9	-96.6	-56.2	+21.1
(1iii) Same par	-21.2	+31.2	+46.5	-57.1	-97.5	-34.0	-149.3
(2i) Main	-74.1	+34.6	+26.3	-65.3	-98.2	-56.4	+14.8
(2ii) Liq targ	-116.1	+31.6	+15.8	-65.2	-97.98	-56.5	+21.1
(2iii) Same par	-19.4	+31.3	+50.2	-59.97	-98.5	-34.2	-149.6
(3i) Main	-44.7	+35.4	+55.5	-72.6	-118.6	-57.4	+14.5
(3ii) Liq targ	-69.9	+32.4	+49.1	-72.3	-120.9	-57.4	+20.8
(3iii) Same par	-8.9	+32.0	+69.9	-69.0	-115.6	-35.3	-150.5
(4i) Main	-65.4	+35.8	-0.0	-64.4	-136.7	-57.7	+14.4
(4ii) Liq targ	-95.6	+32.9	-14.3	-64.2	-141.1	-57.7	+20.7
(4iii) Same par	-25.3	+32.5	+32.3	-59.9	-131.0	-36.1	-150.7

Notes: Difference of percentage changes from initial steady state between baseline and counterfactual as a proportion of the percentage change in counterfactual, multiplied by 100. Changes are cumulative over the two periods of the recession. The first variable represents final output, the next four variables are components of output, and the remaining two variables represent tfp and debt as proxies of aggregate states in the second period. Specification: Arabic numeral (1*) for AR shock with persistence 0.10; (2*) for AR shock with persistence 0.30; (3*) for AR shock with persistence 0.50; (4*) for initial shock fixed over two periods and following AR with persistence 0.30 thereafter. Roman numeral (*i) for counterfactual calibrated to baseline entry rate; (*ii) for counterfactual calibrated to target bankruptcy; (*iii) for counterfactual with same parameters as baseline.

The second row, case (1ii), considers a counterfactual in which the initial steady state is calibrated to match the empirical bankruptcy rate rather than the baseline entry rate. Under this specification, output declines faster in the baseline relative to the counterfactual by 132%. Compared with the benchmark case (1i), the dampening role of firm numbers and entry is weaker, while the amplifying role of indebtedness is stronger. A key reason is that the counterfactual steady state features a lower entry–exit rate—2.37% versus 2.74% in the baseline. As a result, when entry collapses to zero on impact, the baseline experiences a relatively larger reduction in the number of firms operating.

The third row, case (1iii), considers a counterfactual economy with the same parameters as the baseline except for the absence of trade credit. As shown in Section 5, removing trade credit substantially alters the steady state, most notably by lowering firms' debt levels. Under this specification, trade credit amplifies the decline in output by 21%, which is smaller than in the benchmark case (1i). The most notable difference lies in the role of debt. In this case, debt exerts a strong dampening effect through comparatively faster saving (or deleveraging) in the baseline. This reflects the much higher steady-state debt in the baseline, which leaves firms with wider scope to better self-insure by reducing net debt in response to the shock. Despite this dampening force—and the weaker amplifying role of productivity—slower adjustments in input hiring and firm survival still generate substantial amplification. Because the counterfactual implies vast steady-state differences relative to the baseline, however, this specification seems less relevant for identifying the practical role of trade credit.

The next three specifications in Table 11 (cases (2i), (2ii) and (2iii)) consider a higher persistence of the financial shock of 0.30, compared with 0.10 in the benchmark case. The pattern of results across the three counterfactuals remains similar. In particular, trade credit default continues to amplify the recession with an effect of about 74%, which is only slightly smaller than in the benchmark. Because this level of persistence of the shock implies implausibly elevated risk-free lending spreads long after the initial impact, however, these specifications may be less relevant than the benchmark specification.

The next three rows (cases (3i), (3ii) and (3iii)) consider an even higher persistence of the financial shock of 0.50 and yield qualitatively similar results. We do not examine higher persistence values, as they would fail to generate the additional decline in output after the impact period observed in the data.

The bottom rows of Table 11 (cases (4i), (4ii) and (4iii)), consider specifications in which the initial shock is forced to persist for the first two periods, after which it begins to revert according to an AR(1) process with persistence 0.30. Although unconventional, this specification generates a cumulative output decline of -6.5%, closer to the magnitude observed during the Great Recession in the U.S. The main results obtained in the other cases carry over to this specification.

E Appendix: Dynamics

The economy is initially at the baseline stationary equilibrium. Since we are considering the adjustment as a small open economy, the final steady state will differ from the initial one in terms of national debt D and corresponding drop in consumption, a wealth effect. Because of the separability of the utility function, the level of consumption adjusts in the shock period and, because it is a SOE, remains constant thereafter. We must take into account that entry m can be zero in some periods.

The procedure to obtain the transition over periods $t = 0, 1, ..., T$ consists of an outer loop in the price of inputs and new consumption, an inner backward step going from period T to 0 to characterise the firms' and lenders' policy functions and debt price functions, and price of inputs when $m > 0$, and an inner forward step going from period 0 to T to find aggregate quantities, included national debt, and default rates that satisfy the equilibrium conditions, and the price of inputs when $m = 0$.

The steps to finding the equilibrium path are as follows:

1. Guess a long enough time horizon T and consider solving for $t = 0, 1, ..., T$.
2. Guess a path for input prices $\{p_t\}_{t=0}^{T}$.
3. Guess a path for aggregate entry rate $\{m_t\}_{t=0}^{T}$ and consumption $\{c_t\}_{t=0}^{T+1}$.
4. Guess terminal functions W_{T+1} and q_T.
5. Backward loop: Starting at the terminal steady state at $t = T$, for given $\{\rho_t, Q_t\}_{t=0}^{T}$, proceed backwards to obtain sequence of input prices $\{p_t\}_{t=0}^{T}$, and the paths for functions $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=1}^{T}$ and $\{q_t\}_{t=1}^{T}$:

    (a) At t where $m_t > 0$, by solving the equilibrium between final-good firms and lenders and the free entry condition.
    (b) At t where $m_t \leq 0$, by solving the equilibrium between final-good firms and lenders, ignoring free-entry, and leaving the input price unchanged. Unless net value of entry is positive, in which case, look for free-entry price as in 5(a) above, and update to a strictly positive $m_t > 0$.

6. Forward loop: Given the above paths $\{p_t\}_{t=0}^T$ and $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=0}^T$, and some $\{m_t, c_t\}_{t=0}^T$ proceed forwards to obtain the paths $\{\theta_t, w_t, l_t, \mu_{t+1}, N_{t+1}\}_{t=0}^T$, and updated $\{m_t, D_{t+1}, Y_t^N\}_{t=0}^T$ (where Y_t^N is output net of fixed costs) and $\{p_t\}_{t=0}^T$ that satisfy the corresponding equilibrium conditions:
    (a) At t where $m_t > 0$, by taking p_t as given and updating m_t via labour market clearing. Unless the resulting $m_t \leq 0$, in which case, proceed as in point 6(b) below.
    (b) At t where $m_t \leq 0$, by setting $m_t = 0$ and updating p_t via the input wedge condition.
    (c) Debt D_{t+1} satisfies the national resources constraint of period t.
7. Update consumption: Update $\{c_t\}$ at level associated to D_T and Y_T^N via final-goods market clearing.
8. With the updated $\{p_t, c_t\}_{t=0}^T$, back to step 5.
9. Check that T is long enough. Update T and back to step 2.

Further details of the two loops in steps 5 and 6 are available upon request and in the code provided.

Implementation for no-trade-credit model: The counterfactual analysis consider models without trade-credit default. The procedure has to be adapted as follows. In the forward loop, for $m_t > 0$ the input pricing condition is becomes $w = p$. In the forward loop, for $m_t = 0$ the input pricing condition becomes $p = w$.