Trade Credit Default*

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May 22, 2024

Abstract

Default on trade credit repayments is substantial, about 7% in the U.S. This paper studies the role of trade-credit default in the transmission of macro shocks. We present cyclical facts about trade credit and its defaults. We then build a heterogeneous-firms quantitative model where an intermediate input is purchased by final-goods producers partly on trade credit before observing the realisation of their productivity. Trade credit's role stems solely from its low seniority status. Aggregate trade-credit default is priced by input suppliers; individual bankruptcy risk is priced in by lenders supplying bank credit. A markup effect and an insurance effect are two novel mechanisms behind our findings. First, in reflecting trade-credit default spillovers, endogenous shifts in the markup charged by intermediate input suppliers contribute materially to the size of fluctuations. Second, via the interplay of the markup and insurance channels, trade-credit default amplifies the impact of financial shocks and TFP shocks, but dampens that of volatility shocks.

Keywords: trade credit, default, delinquency and bankruptcy, entry and exit, heterogeneous firms, transmission and amplification of shocks, endogenous markups

JEL No. D21,D25,E32, E44, G33

1 Introduction

Default on trade credit payments is sizable. Jacobson and von Schedvin (2015) report that trade creditors experience significant losses due to failed payments, a channel of transmis-

*Earlier versions of the paper had been circulated under the different titles *Trade credit delinquency, bankruptcy, and the propagation of aggregate shocks* and *The role of trade credit and bankruptcy in business fluctuations*. This paper has been presented at the AEA 2019 Conference in Atlanta, the National University of Seoul, University of Kent, III Workshop Spanish Macroeconomics Network, Universität of Konstanz and the London Macro CfM LSE 2021, SED 2022 and U of Manchester, and we much appreciate the feedback from audiences there. Special thanks to Dean Corbae and Gabriel Michalache for their helpful comments on an early version, and to Stephen Terry and Jordi Gali for enlightening conversations and suggestions. Aizat Token and Gabriel Bracons provided superb research assistance. The authors would like to acknowledge financial sup-

sion of financial disruptions from firm to firm, and Amberg, Jacobson, and von Schedvin (2020) provide evidence that firms charge premia in transactions involving trade credit. For the U.S., based on inter-firm credit sales data, Costello (2020) reports a very substantial fraction of receivables that are past due, and establishes the importance of this form of trade credit deterioration in the transmission of liquidity shocks. While this evidence suggests that trade credit default might be consequential at the micro level, not much is known about its quantitative importance in the transmission of macroeconomic fluctuations. In effect, trade credit default has been thus far absent in the macroeconomics literature.[1]

Objective and preview - In the first part of the paper we provide motivating empirical evidence of the macroeconomic patterns of trade credit and trade-credit default. Using both aggregate and micro data, we find that while trade credit is procyclical, the ratio to GDP or sales is acyclical, and that default is countercyclical. Our contention is that trade-credit default brings forth novel mechanisms of significance for macroeconomic outcomes. In the rest of the paper, we thus set out to investigate the determinants of trade credit default jointly with bankruptcy and liquidation, and its implications for macroeconomic variables. The specific objective is to assess the quantitative contribution of trade-credit default to fluctuations in GDP and employment, and identify the various channels at play. Using a dynamic model of heterogeneous firms with endogenous credit risk, we identify a *markup* effect and an *insurance* effect as two novel mechanisms. We first find that, in reflecting trade credit default, endogenous shifts in the markup charged by intermediate input suppliers materially contribute to the transmission of aggregate shocks. Second, via the interplay of the markup and insurance channels, trade credit default amplifies the impact of financial shocks and productivity shocks, and dampens that of volatility shocks.[2]

Model - We build a quantitative general-equilibrium heterogeneous-firms model consisting of four types of agents: a representative intermediate-goods supplier that uses labour to produce an intermediate input; heterogeneous final-good producers that use the input from intermediate suppliers to produce a final good; households who act as consumers, shareholders, bond holders and workers; lenders/banks that take deposits from households and lend to final-goods firms. Some proportion of inputs are purchased on within-period trade credit, meaning that these inputs are delivered at the beginning and due to be paid at the end, only after productivity shocks to final-goods firms are realised. Final-goods firms may also hold non-contingent bank debt or save in liquid assets. Borrowing and savings will be responding mainly to financial commitments and precautionary motives, but they will also be influenced by an agency problem along the lines of Arellano, Bai,

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[1] Including, for instance, the vast body of work emphasising the role of financial frictions in fluctuations since the seminal Bernanke, Gertler, and Gilchrist (1999) and Kiyotaki and Moore (1997b).

[2] These shocks are instances of driving factors considered in the rapidly growing literature on aggregate fluctuations with firms which, without being comprehensive, includes Jermann and Quadrini (2012), Khan and Thomas (2013), Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), and more recently Khan, Senga, and Thomas (2016), Arellano, Bai, and Kehoe (2019) and Ottonello and Winberry (2020).

and Kehoe (2019). Final-goods firms in operation cannot issue new shares so dividends must be non-negative; we also assume non-positive dividends if there is bankruptcy or trade-credit delinquency. Trade credit debt is junior to bank credit debt, consistent with legislation and practice.[3] Its junior status will be the fundamental reason why trade credit matters in this model.

Liquidation by a final-goods firm means exit and possibly bankruptcy, and will imply default on trade credit. A firm can also default on trade credit while honoring their other financial commitments in what we call delinquency. Delinquency implies a loss of future output to the firm over a random number of periods. Final-goods firms are subject to idiosyncratic productivity shocks. These shocks are observed only after inputs are purchased. Because of this timing, final-goods producers might be unable to honor all of their financial obligations (i.e., they become liquidity constrained). The aggregation of default on trade-credit payments results in a trade-credit loss rate that is factored in the intermediate-producer's pricing of inputs. On the other hand, an individual firm's bankruptcy risk is reflected in the lending rates offered by banks.

Discussion of Model Assumptions - The fraction of inputs sold on trade credit will be exogenous in this model, an assumption that is well justified given our focus on short term fluctuations and the acyclical pattern seen in data. A sensible model endogenising this variable would have to imply at best a weak response either way. As noted below, this is also a common assumption in the literature.[4] The assumption that input suppliers pool risk from final-goods producers can be well justified as well. Trade credit contracts are standarised at the industry level, a fact documented and used in Klapper, Laeven, and Rajan (2012) to defend the same assumption. This observed practice would emerge in an environment where suppliers do not observe a buyer's characteristics beyond their order's size. Even if the seller could infer repayment risk from order size, price differentiation based solely on the amount purchased would not stand before US competition law. The Robinson-Patman Act, 15 US code §13, forbids price discrimination, and contemplates companies adjusting terms, including denial of credit, but only on reasonable standards of credit worthiness. The observed industry practice of uniform pricing suggests that non-financial firms have only limited capacity to observe and act on reasonable credit worth characteristics. Our assumption reflects this view.[5]

Mechanisms - This model contains two novel mechanisms. The first is one whereby trade-credit default, stemming from both delinquency and liquidation of final-goods firms, im-

[3]This is a fact well acknowledged and documented in the literature, e.g., Cuñat and Garcia-Appendini (2012) and Jacobson and von Schedvin (2015), resting under the legal protection of contractual subordination provisions contained in bank loan covenants, e.g., Bratton (2016). The junior status of trade credit is itself the subject of research, e.g., Longhofer and Santos (2003), Zhang (2019) and Garcia-Appendini and Montoriol-Garriga (2020).

[4]This constant fraction of trade credit sales then renders largely irrelevant the price differentiation between cash and credit inputs since they are sold as a bundle. For ease of exposition, we assume away such price differentiation. Extending the model in that direction would, if anything, strengthen the transmission mechanism. Further discussion later and in Appendix B.1.

[5]Explaining the observed practice as an endogenous outcome in a firm-to-firm relationship setting would be an interesting problem but, based on the above, it is not warranted as a first step towards the question of this paper.

poses losses directly on intermediate-input firms which requires a rise in the markup of the price of inputs over the unit labour wage cost. This general-equilibrium 'markup' effect calls for some mix of lower wages and higher input price, the former discouraging labour supply and the supply of inputs, and the latter imposing higher operating costs to the other final-goods firms that are conducive to a reduced demand for inputs and hence employment and to further bankruptcies and defaults. Thus, if correlated with the macroeconomic shocks driving fluctuations, this markup channel will contribute to recessions and fuel further bankruptcies.

There is a second distinctive mechanism which we label 'insurance' since delinquency helps avert risks of costly liquidation. Its implications are double-edged. One one hand, the option to choose delinquency, in order to avoid liquidation and exit, induces increased firm's demand for inputs. Indeed, when a poor idiosyncratic productivity realisation renders unfeasible full repayment of the inputs purchased, liquidation carries costs that include the lost continuation value of the firm. Delinquency avoids these costs and thus hedges some of the risk associated with larger input purchases, making them comparatively more desirable. On the other hand, this element of insurance also means weaker precautionary motives and firms holding lower buffer stocks of savings and higher debts. All in all, in the face of aggregate shocks, the insurance channel may mitigate recessions by smoothing the drop in demand for inputs but, on the other hand, can also lead to more exit, and less entry, and hence deeper recessions due to insufficient self-insurance.

Calibration and Results - Parameter values are chosen so the model in the stationary equilibrium matches a number of empirical targets based on U.S. firm-level and aggregate evidence regarding firms' indebtedness, operating profits, trade credit, trade-credit default, and bankruptcy. The calibrated model is able to deliver a very good approximation to the data targets. The calibrated model also does reasonably well regarding some non-targeted variables.

To address the question of this paper, we study the dynamic response of the economy to various aggregate shocks. We consider financial, volatility, and total factor productivity shocks. To discipline the analysis, we choose the shocks so the baseline model produces a response that best approximates the fall of GDP in the U.S. during the Great Recession as measured in Arellano, Bai, and Kehoe (2019). We conduct two types of exercises. We are first interested in the role of the markup channel in the transmission of the impact effect of the shock. As already outlined, the transmission must work through a widening of the markup in the form of lower wages or higher cost of inputs. We therefore compare the impulse-response impact to that of an economy where the markup default premium remains unchanged along the transition. There is generally a sizable role of the markup in the impact response of output and employment, as well as defaults, to those shocks, accounting for over two fifths of the impact of a financial shock and one fourth for a volatility shock, and a much smaller part for a TFP shock.

Second, we study the amplification effects of trade-credit default by comparing the response to the shocks in the baseline model with that in a counterfactual model without trade-credit default. While the markup channel must work towards amplification, the role of the insurance mechanism, through its effects on both the demand for inputs and firms' exit and entry, is generally ambiguous. Under the financial shock, trade credit default amplifies the impact response of output by about 10%, a result driven mainly by the sharper rise of exit in the baseline model because of the insurance channel. Under the volatility shock, in contrast, the amplification effect is strongly negative, in the order -30%, mainly a consequence of the much more muted fall in input demand in the baseline model than in the counterfactual due, again, to the insurance channel of trade-credit default.

Contribution to Literature - We contribute to the literature at various levels. We address the question of the role of endogenous trade credit default in macroeconomic fluctuations. The recent, and notable, papers that study trade credit default, Boissay and Gropp (2013), Jacobson and von Schedvin (2015) and Costello (2020), are empirical micro studies. The seminal Kiyotaki and Moore (1997a)'s interest is in the transmission of defaults through credit chains at the micro firm level, an idea that Boissay (2006) extends to look at macro implications but in a static partial-equilibrium setting. The emerging literature on trade credit (e.g., Altinoglu (2021) and Luo (2020)), including those with a more macro leaning (e.g., Reischer (2020) and Bocola and Bornstein (2023)) do not study default. A second contribution is the evidence on the subject, and especially the measure of losses on trade receivables based on firm-level data, that we use for motivating the paper and for supporting assumptions and the quantitative analysis. Some literature have also looked at cyclical patterns albeit with a different focus and sampling choices, e.g., Reischer (2020) and Boissay (2006), where only the latter have considered measures of losses to trade credit.

The third type of contribution is methodological, related to the modeling. The representation of trade-credit default and thus the markup and insurance mechanisms that we identify are unique to our model. One strand of literature on trade credit, including Altinoglu (2021), Luo (2020) and Reischer (2020), usefully highlights the role of financial chain linkages through networks but within an otherwise static setting that omits defaults. As in our paper, trade credit is exogenous in these papers, or an exogenously specified functional relationship with credit spreads in the case of Reischer (2020). Unlike these papers, we have a fully specified intertemporal model where spreads and financial conditions are endogenously determined reflecting equilibrium repayment risks. We acknowledge that the network effects we set aside may be a relevant channel, but it would be in addition, not an alternative, to the transmission effects we study. After all, our markup effect captures spillovers of a similar nature to the amplification effects within the credit chains of, e.g., Kiyotaki and Moore (1997a).

With a more characteristically macro bend, in Bocola and Bornstein (2023) the amount of trade credit, represented by long-term relationships, is endogenously determined by a reputational constraint that enforces repayments and thus rules out defaults. In our model of anonymous decentralised trade, a similar lack of commitment shows in aggregate markups, repayment losses and the occurrence of defaults, the variables we are therefore able to study anew here. Our trade-credit share of sales is exogenous (reasonably, given our purpose and evidence), and Bocola and Bornstein (2023) analogously postulates an exogenous fraction of input purchases that need financing, with within-period bank credit plugging the gap when trade credit is insufficient. In our decentralised equilibrium there are no reputational constraints and firms can and do finance that fraction on inputs entirely via trade credit, while it is intertemporal (rather than within-period) bank credit that might be used to cover cash flow needs including also pre-existing debt obligations and operating costs. In Bocola and Bornstein (2023), bank credit limits ensure full repayment via a static enforcement condition; in our case, debt prices and borrowing limits are forward-looking as they factor in positive liquidation risk in equilibrium. That paper does not seem to consider entry and exit, while they are important for the transmission of shocks in our model. As a final note on the broader trade credit literature, while they all assume some friction that confers a special role to trade credit, in our model we are offering a theory that micro-founds the role of trade credit based solely on its low seniority status.

Our paper also relates to models in the strand of quantitative macroeconomics with heterogeneous firms and bankruptcy risk that includes Arellano, Bai, and Kehoe (2019), Khan, Senga, and Thomas (2016) or Ottonello and Winberry (2020). In these papers, default risk is important for determining a firm's credit access, i.e., borrowing constraints and rates, but the realised default should be regarded as primarily a consequence rather than a cause or amplification factor of macro fluctuations. Indeed, since debts are priced individually, firms' default risk would not have in and by itself much independent aggregate impact. Our model certainly embeds this mechanism of default-risk pricing of bank debt too,[6] but importantly includes new additional and fundamentally different mechanisms. The insurance mechanism operates precisely through bankruptcy and exit, becoming directly relevant for this literature. Regarding the markup mechanism, default on trade credit has aggregate consequences via the spillover effects coming through the pricing of intermediate inputs. This stands in contrast with the existing models based solely on defaults on bank debt. Notably, note that in our model, besides resulting from delinquency, trade-credit default also follows from the event of firm bankruptcy/liquidation. Firm bankruptcy risk will therefore be reflected in intermediate-input markups. It is thus that our model incidentally brings forward a novel macroeconomic relevance for bankruptcies and represents a meaningful contribution to that literature.

The fourth type of contribution is in the specific results. Our aggregate shocks to firms' liquidity position relate to the financial shocks that have attracted considerable macro research, e.g., Jermann and Quadrini (2012), Khan and Thomas (2013) Khan, Senga, and Thomas (2016) or, more recently, Mehrotra and Sergeyev (2021), and our finding that trade-credit default is important in the transmission is therefore informative to that literature. The trade credit literature has also focused on financial shocks, e.g., Altinoglu (2021) and Reischer (2020), and, compared specifically to Bocola and Bornstein (2023), the smaller amplification from trade-credit we find reflects the interplay of our novel effects associated with its default. In another recent literature that includes Arellano, Bai, and Kehoe (2019), Bloom et al. (2018) and Alfaro, Bloom, and Lin (2024), uncertainty shocks

[6]While our model shares these paper's simple representation of bankruptcy as liquidation, Corbae and D'Erasmo (2021)'s focus is on the finer elements of the actual bankruptcy code.

play an important role, and our finding that the presence of trade-credit default dampens the impact of volatility shocks is thus relevant to that conversation. Finally, the material role that here markup variation plays in the transmission of fluctuations bears on the mainstay of the DSGE literature where countercyclical markups are a core theme, e.g., Rotemberg and Woodford (1999) for an early review.[7] We provide a novel mechanism for a meaningful role of endogenous countercyclical markups which is yet separate from the nominal rigidities and/or imperfect competition elements in that literature

In the remainder of the paper, section 2 documents empirical evidence, section 3 presents the model and outlines mechanisms, section 4 shows the calibration and evaluates it, sections 5 analyses the transmission of shocks and section 6 the amplification from trade-credit default. Section 7 concludes.

2 Empirical evidence

This section presents empirical measures of trade credit and trade-credit default, and their cyclical behavior. It will serve the double purpose of documenting basic patterns of variables subject of our analysis and also providing empirical backing for some of our modeling choices.

We draw on aggregate quarterly data from the Federal Reserve Board and the U.S. Bureau of Economic Analysis, and individual firm annual data from Compustat, for the period 1980-2016. Besides GDP and the price deflator, we use data on business sales (turnover) and employment, trade receivables accounts, bank charge-off rates and, importantly, receivables estimated doubtful. Doubtful accounts is the amount of all current accounts receivable estimated to be uncollectable and will be used to construct measures of trade-credit default. We seem to be among the few, apart from Boissay (2006), to present information about Compustat doubtful accounts. Definitions, data sources and details of the variables constructed are in appendix A.1 for aggregates, and in A.2 for firm-level data. Since the emphasis of this paper is on the implications of defaulted payments to input suppliers, we consider the evidence related to account receivables. The main summary statistics are in Table 1.

Aggregate data - We start with the aggregate data from U.S. Financial Accounts and NIPA. We construct the time series of the ratio of trade credit receivables to GDP over the period since 1980 and find that the trade-credit-to-GDP ratio has stayed consistently around 18% throughout, with no apparent trend. This would be confirmed with the series extended further back to 1960. This ratio obviously varies at high frequency but, except for a spike around year 2000, appears to remain within a fairly narrow band.

Considering now cyclical properties of trade credit, the 0.52 correlation between the log changes in GDP and in trade credit receivables shows that trade credit is procyclical, albeit

[7] See Bils, Klenow, and Malin (2018) and Nekarda and Ramey (2020) for recent empirical investigation, and Jaimovich and Floetotto (2008) in the context of firm dynamics.

Table 1: **Summary Statistics**

Aggregate data		*Firm-level data*	
Mean TC to GDP	0.18		
		Mean TC loss rate	0.07
Corr GDP growth with:		Corr Sales growth with:	
TC growth	0.52	TC growth	0.57
TC/GDP change	0.07	TC/Sales change	-0.16
		TC-loss-rate change	-0.43

Note - Summary statistics for trade credit and loss rate. TC denotes trade credit. Data sources: Federal Reserve Board, BEA and Compustat, 1980-2016. See Appendix A.1 and A.2 for definitions and construction of variables. Authors' own calculations.

not as strongly as the common main macroeconomic aggregates. On the other hand, the correlation between the log change in GDP and the change in the ratio of trade credit to GDP is a very weak 0.07. This evidence renders this ratio as an acyclical variable, one which does not follow systematically business cycle fluctuations in either direction. When studying cyclical shocks, it is therefore empirically plausible that trade credit as a share of output does not comove systematically. Our modeling assumptions will reflect this observation.

Firm-level data - We turn now to firm-level Compustat data. The aim is to construct and study variables that somehow can be compared to the aggregates just discussed. From the individual firm data, we have constructed yearly aggregates of trade credit receivables and sales. The cyclical behaviour of trade credit receivables can thus be described by the correlation of its log change with the log change in total sales, at yearly frequency this time. The 0.57 correlation is comparable to the figure obtained on aggregate data, portraying trade credit as procyclical, although not to a particularly strong degree. Regarding the ratio of trade credit to sales, the -0.16 correlation coefficient between changes in this variable and the log change in sales turns out to be quite small in size, and negative in this instance. This observation would confirm the result already obtained from aggregate data of an economy's share of trade credit that is acyclical.

We turn now to trade credit default, a type of variable unavailable on aggregate time series. Based on the Compustat firm-level variables for doubtful accounts receivable and trade credit receivables, we construct firms' ratios of doubtful accounts to receivables as a proxy of trade-credit default, or expected trade-credit loss, for suppliers. We obtain aggregate yearly observations as the average of this measure of the trade-credit loss rate across firms within each year. The average ratio of receivables doubtful over receivables, doubtful or not, gives an empirical loss rate on trade credit of 6.6% over the period. (This rate is not far from the estimate in Jacobson and von Schedvin (2015) of 8% for Sweden based on direct administrative data.) As shown in Fig. 1, the time series shows two visible spikes, a sharp one coinciding with the early 2000's dotcom bust, and a smaller one during the 2008

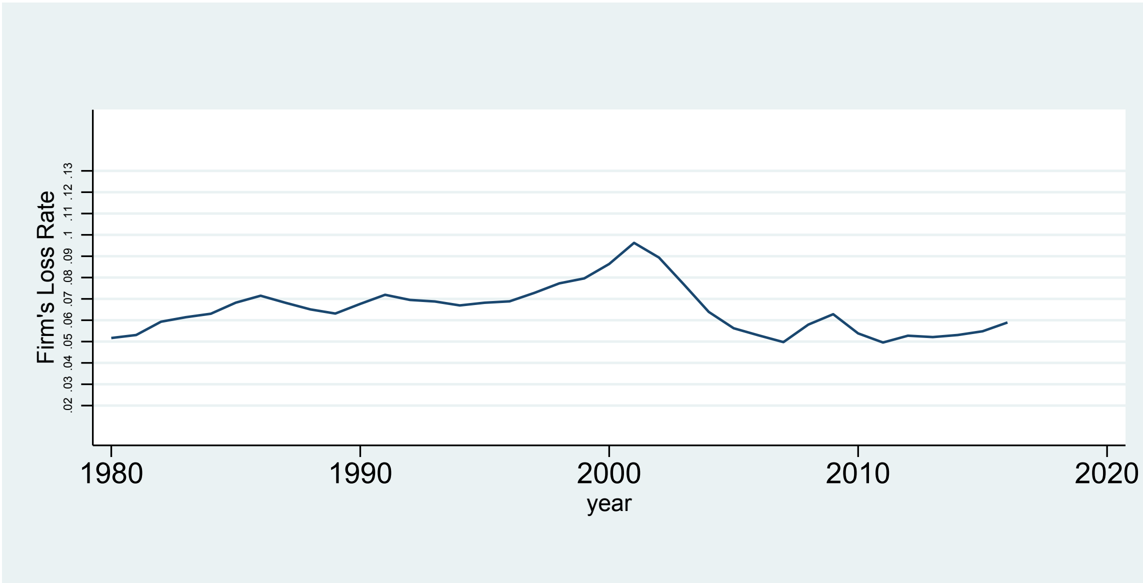

Figure 1: Trade credit default rate.

Note - Time series of trade credit default. Source: Compustat 1980-2016. Year average firm-level trade-credit loss: doubtful accounts receivable to trade credit receivables. See Appendix A.2. Authors' calculations.

financial crisis. Regarding its cyclical properties, at annual frequency, the correlation of changes in the trade-credit loss rate and sales growth has a negative coefficient of -0.43. The loss rate thus appears to be countercyclical.[8]

Summing up, the ratio of trade credit to output is acyclical in aggregate Fed time series, a fact that is supported by the aggregation of firm-level data, and the trade credit default risk loss rate is countercyclical and stands at about 7%.

3 Model

The model contains four types of agents: producers of intermediate inputs, producers of final goods, financial intermediaries, and households. Firms are all competitive, and inputs and goods are homogeneous. Final-goods firms face constraints on the issuance of shares, and experience idiosyncratic shocks. They pay for some inputs with a delay, only after shocks are realised. Since we assume there is no commitment to repayments on this within-period trade credit, these firms may fail to pay in full their suppliers of inputs and may thus become delinquent. Final-goods firms can also save with or borrow from

[8] For comparison, we can consider a measure of losses associated with bank loans to firms. Based on aggregate data from Fed described in Appendix A.1, charge-off rates on business loans are only 0.78% on average, and countercyclical, with a correlation of changes in charge-off and GDP growth of -0.25.

banks, with spare borrowing capacity subject to an agency problem. These firms may also liquidate and declare bankruptcy. We introduce next notation and the assumptions of the model, and will wait until later to set out the formal details in recursive form and, finally, discuss the mechanism of interest.

3.1 Assumptions

Input producers - There is a continuum of intermediate producers with mass 1. The intermediate input x is produced from labour n on a one-for-one basis so that $x = n$.[9] The price of the input is p.[10] Payments from retail customers are received with a delay within the period. A given proportion of sales τ is on trade credit and a proportion of those, given by the trade-credit default, or loss, rate θ, will not receive payment. The remaining fraction of sales $1-\tau$ are on cash and receive payment for sure. Section 1 had already argued that τ being exogenous and θ pooling the default risk across all retailers are both very sensible assumptions.

The cash-flow to the representative intermediate input firm includes the costs of labour at the wage rate w, and it becomes $px - \theta\tau px - wx$. There is free entry in the input producing sector. The rate of default on trade credit sales θ is forecast at entry. Free entry thus implies zero profits given the information available at the beginning of the period: $px - \theta\tau px - wx = 0$. The markup of the input price over the wage rate, w/p, is thus determined by the default forecast θ.

Final-goods producers - Output from a final-goods firm, y, depends on aggregate productivity z, idiosyncratic productivity ϵ, and the amount purchased of intermediate input x through a function $F(.)$ according to $y = ze^{\epsilon}F(x)$. The idiosyncratic productivity is stochastic and follows a Markov chain with transition probabilities $\psi_\epsilon(\epsilon'|\epsilon)$ over a support $\mathcal{E}$. There is free entry of new firms and an entry cost ξ^E which must be paid by issuing shares. After paying this cost, the firm draws a realisation of the initial idiosyncratic shock ϵ_{-1} from an initial distribution given by its stationary distribution $\overline{\psi}_\epsilon(\epsilon_{-1})$. There is a fixed cost of operating the firm c_F in every period. This cost may include, for example, the replacement of capital. It will include the manager's wage w_m, equal to an exogenous outside option.

At any given period, the firm chooses the amount of intermediate input x before the realisation of the shock. After the shock is observed, the firm can issue one-period debt $b > 0$ at a discount price q, or save cash assets, that is negative debt $b < 0$, at a discount price Q_0, decide whether to repay input suppliers by choosing $d^x \in \{0, 1\}$, and whether to liquidate and hence, if $b > 0$, declare bankruptcy on bank debt, $d^b \in \{0, 1\}$. Delinquency on input payments $d^x = 1$ implies a loss of a proportion of output $\tilde{\nu} > 0$ in future periods. The delinquency indicator is $\nu \in \{0, \tilde{\nu}\}$. If $\nu > 0$ the probability of forgiveness and having

[9]Our model might seem isomorphic to a model where labour is the intermediate input. But that is not the case: (1) Wage commitments are senior to any other liabilities; (2) would require unrealistic highly centralised wage bargaining; and (3) our data in on trade credit receivables, and default on wages is too rare in practice (see also Arellano, Bai, and Kehoe (2019) on this point).

[10]See Appendix B.1 for discussion of the uniform price for all inputs.

the penalty cleared, conditional on not incurring further delinquency, is λ. Liquidation $d^b = 1$ leads to bankruptcy if $b > 0$ and exit.

Under bankruptcy or delinquency, claimants (i.e., creditor banks or trade-credit suppliers) receive the residual value of the firm; recovery r^b goes to banks and r^x goes to trade-credit suppliers. Because of the fixed cost c_F and the cash input payments $(1-\tau)px$, the residual value may be negative in which case the firm does not pay the fixed cost and ceases to operate but still pays the cash inputs.

The firm maximises the expected discounted value of dividends. The discount rate is ρ to be determined as the household's equilibrium discount factor. A firm faces the constraint that it cannot issue new shares so dividends cannot be negative during the life of an operating firm.[11] However negative dividends may occur for firms that are liquidating and, additionally, cease to operate since these firms must still meet the cost of cash inputs. A firm is also constrained not to pay positive dividends or retain profits (i.e., save cash) when liquidating or becoming delinquent. Finally, debt has seniority over trade credit sales: trade credit recovery cannot happen before all debts have been repaid. In Section 1 above we have already justified the basis for this seniority ranking. The preceding description means also that cash sales are senior to bank debt.

Finally, the firm faces an agency problem. As said, the firm employs one manager. The manager lives for one period (i.e., we are in the myopic case as in Arellano, Bai, and Kehoe (2019)) and, because of lack commitment, may have incentives to divert at the end of the period some firm's liquid capacity. If there is no diversion, the manager earns the wage w_m at the firm. Alternatively, the manager can seize liquid funds available within the firm to purchase cash inputs and employ them into a side project and extract a profit using the same production function as in the current firm but scaled by a wedge factor η. The liquid funds available for diversion consist of the sum of the firm's saved cash, if any, plus the spare unused available bank credit. This agency problem means the firm will seek to contain the spare liquidity buffer to preempt diversion, a participation constraint.

Financial intermediaries - Lenders extend one-period loans to final goods firms. They have the same information that is available to firms, so there is a contract for each type of loan in terms of size and characteristics of the firm. Competition drives the surplus for lenders on all loan types to zero. In this way, the discount prices of debt q reflects the default risk— implied by the firm's decision d^b and recovery r^b — over the market discount price Q. These lenders fund their loans by selling securities to households at discount price Q.

Intermediaries hold cash deposits of firms at discount price Q_0, and use the funds to purchase risk-free securities at the market discount price Q. There is an intermediation cost which, under free entry competition in intermediation, drives a wedge, or spread, $q_{\text{spr}} \geq 1$ between the firms cash savings and the market rate of return so $Q_0 = Q \times q_{\text{spr}}$.

Households - Households own the firms and the labour force. There is a representative infinitely-lived household who can borrow and lend freely at the discount price Q. Households, with subjective discount rate β and period utility $u(c, l)$, supply labour l to input

[11] Here we follow Arellano, Bai, and Kehoe (2019) and Khan, Senga, and Thomas (2016).

producers and consume c optimally.

Equilibrium - An equilibrium satisfies a set of conditions: Decision rules for borrowing/saving and repayments maximise final-goods firms' objective given debt prices and input price; the price of inputs is such that intermediate producers make zero profits given aggregate trade-credit loss and wages; the wage and mass of final-goods firms is such that the labour market clears and there is free entry; prices of loans satisfy zero profit for lenders given the decision rules of final-goods firms; exit flows reflect optimal decisions; households choose consumption and supply labour optimally; the distribution of firm's types is consistent with the above. We consider a small open economy that takes the Q as given, with external debt denoted D.

3.2 Recursive representation

We proceed to cast the model more explicitly in recursive form. The aggregate state at the beginning of a period, S, includes the distribution $N \times \mu$ of firms over characteristics (ϵ_{-1}, b, ν) at the start of the period, the claims held by the representative household A, and external debt D: $S = (N \times \mu, A, D)$. This distribution of firms consists of a probability measure μ scaled by the mass of firms N. The law of motion for the aggregate state is $S' = H(S)$, with its components denoted $N' \times \mu' = H^{\mu}(S)$, $A' = H^{A}(S)$, and $D' = H^{D}(S)$. As a fraction of N, the level of entry of new firms can be written $m(S)$. The input price, wage rate and trade-credit loss rate can therefore be written as $p(S)$, $w(S)$ and $\theta(S)$. The final-goods firm's individual state before shocks are realised consists of (ϵ_{-1}, b, ν). After shocks are realised, in the second part of the period, the individual state becomes (ϵ, b, ν, x), which includes the level of input x chosen in the first part of the period. The discount price of bank debt is a function $q^{ND}(b', \epsilon, \nu|S)$ if there is currently no delinquency, and $q^{x}(b', \epsilon|S)$ if there is delinquency. Given the world Q, the discount rate of the firm ρ, and the price of firms' cash deposits Q_0 will be scalars.

In the rest of this section, we spell out elements needed to make operational the definition of equilibrium given in section 3.1. Appendix C.1, using additional model details from appendix B, will contain the formal definition.

3.2.1 Input producers

Given $\theta(S)$, $p(S)$, and $w(S)$, following the discussion in section 3.1, zero profits in the production of inputs means that the ratio of price of inputs to the wage rate satisfies

$$\frac{p(S)}{w(S)} = \frac{1}{1 - \theta(S)\tau}. \tag{1}$$

This markup depends positively on the trade-credit default loss rate θ. This will be the key relationship for the macroeconomic role of trade-credit default through pricing.

3.2.2 Final-goods firms' decisions

There are two stages to the firm's problem. Denote by $V(\epsilon, b, \nu, x|S)$ the value function in the second stage, after the realisation of the shock, and by $W(\epsilon_{-1}, b, \nu|S)$ the value in the first stage before the current shock are observed. In the second stage, the decision needs to evaluate the value from 3 different courses of action: repayment, delinquency and liquidation.

Repayment - When honoring all obligations, borrowing/savings b' solves

$$\begin{aligned}
V^{ND}(\epsilon, b, \nu, x|S) \quad &= \quad \max_{b' \in \mathbb{R}} \Big\{ \text{cih}(\epsilon, \nu, b, x|S) + q(b', \epsilon, \nu|S)b' + \rho \mathbb{W}'(\epsilon, \nu, b'|S) \Big\} \qquad (2) \\
\text{s.t.} \quad & \text{cih}(\epsilon, \nu, b, x|S) \equiv (1-\nu) z e^{\epsilon} F(x) - c_F - p(S)x - b, \\
& q(b', \epsilon, \nu|S) \equiv \begin{cases} q^{ND}(b', \epsilon, \nu|S) & b' \geq 0 \\ Q_0 & b' < 0, \end{cases} \\
& \mathbb{W}'(\epsilon, \nu, b'|S) \equiv \mathbf{I}_{\nu > 0} \Big(\lambda W(\epsilon, b', 0|S') + (1-\lambda) W(\epsilon, b', \tilde{\nu}|S') \Big) \\
& \qquad\qquad + \mathbf{I}_{\nu = 0} W(\epsilon, b', 0|S') \text{ where } S' = H(S), \\
& \text{cih}(\epsilon, \nu, b, x|S) + q(b', \epsilon, \nu|S)b' \geq 0, \\
& \mathbf{I}_{b' < 0} Q_0 \, (-b') + B^*(\epsilon, \nu) - \mathbf{I}_{b' \geq 0} q^{ND}(b', \epsilon, \nu|S)b' \leq M(\epsilon, \nu), \\
& B^*(\epsilon, \nu) \equiv \max_{b' \geq 0} q^{ND}(b', \epsilon, \nu|S)b', \\
& \eta (1-\nu) z e^{\epsilon} (M(\epsilon, \nu)/p(S))^{\gamma} = w_m.
\end{aligned}$$

The first three conditions define cash in hand cih, the price q of borrowing or cash savings, and the expected continuation value $\mathbb{W}'$. The fourth condition is the constraint that dividends cannot be negative. The fifth condition is the participation constraint: the total amount of liquid resources left at the end of the period (the left hand side) cannot exceed the level that would induce diversion by the manager $M(\epsilon, \nu)$ (the right hand side). The next two conditions define $B^*(\epsilon, \nu)$ as the total credit line available to the firm, and $M(\epsilon, \nu)$ as the level of funding that makes the manager indifferent between the profits from the side project, with productivity parameter η, and the wage at the firm w_m. Details of the manager's problem are in Appendix B.2. The solution gives decision rule $b' = g^{ND}(\epsilon, b, \nu, x|S)$, and the value of dividends

$$\begin{aligned}
\pi^{ND}(\epsilon, b, \nu, x|S) &= (1-\nu) z e^{\epsilon} F(x) - c_F - p(S)x - b \\
&+ \begin{cases} q^{ND}(g^{ND}(\epsilon, b, \nu, x|S), \epsilon, \nu|S) g^{ND}(\epsilon, b, \nu, x|S, z) & \text{if } g^{ND}(\epsilon, b, \nu, x|S, z) \geq 0 \\ Q_0 g^{ND}(\epsilon, b, \nu, x|S, z) & \text{otherwise} \end{cases} \qquad (3)
\end{aligned}$$

Delinquency - When repudiating payments for trade-credit input supplies, the firm determines borrowing b' and the supplier's recovery r^x according to

$$
\begin{aligned}
V^x(\epsilon, b, \nu, x|S) \quad &= \quad \max_{b', r^x \geq 0} \Big\{ \text{cih}(\epsilon, \nu, b, x|S) + q^x(b', \epsilon|S)b' - r^x + \rho \mathbb{W}'(\epsilon, b'|S) \Big\} \qquad (4) \\
\text{s.t.} \quad & \text{cih}(\epsilon, \nu, b, x|S) \equiv (1-\nu) z e^{\epsilon} F(x) - c_F - (1-\tau)p(S)x - b, \\
& \mathbb{W}'(\epsilon, b'|S) \equiv W(\epsilon, b', \tilde{\nu}|S') \text{ where } S' = H(S), \\
& b' \geq 0, \\
& \text{cih}(\epsilon, \nu, b, x|S) + q^x(b', \epsilon|S)b' - r^x = 0, \\
& \mathbf{I}_{b'<0} Q_0 \; (-b') + B^*(\epsilon) - \mathbf{I}_{b'>0} q^x(b', \epsilon|S)b' \leq M(\epsilon, \nu), \\
& B^*(\epsilon) \equiv \max_{b' \geq 0} q^x(b', \epsilon|S)b', \\
& \eta(1-\nu) z e^{\epsilon} (M(\epsilon, \nu)/p(S))^{\gamma} = w_m.
\end{aligned}
$$

The first two conditions define cash in hand cih given now there is trade credit default, and the continuation value $\mathbb{W}'$ given the resulting delinquency flag. The third condition is the constraint that rules out retained profits under default. The fourth condition reflects that dividends, while being non-negative, have also to be non-positive under delinquency. The fifth condition is the manager's participation constraint. The next two conditions define again $B^*(\epsilon)$ and $M(\epsilon, \nu)$ as the total credit line available and the maximum level of funding that enforces manager's participation. The solution gives borrowing $b' = g^x(\epsilon, b, \nu, x|S)$, and the residual recovered $r^x = r^x(\epsilon, b, \nu, x|S)$.

The outcomes under full repayment and delinquency in eq. (2) and (4) admit a sharp characterisation. Consider first the option of full repayment represented by (2). It is useful to begin supposing there is no agency problem (i.e., $\eta \to 0$). A helpful property is that a firm will borrow at most as much as needed to meet its financial commitments, not more. Borrowing to pay dividends is not optimal.[12] A firm with positive cash in hand would therefore never borrow, and would choose to pay some dividends, and also keep some cash reserves for precautionary motives. A firm with negative cash in hand, on the other hand, will borrow just to keep repayments, that is, insofar as credit constraints make it feasible. Introducing the agency problem will work towards the firm, relative to the commitment case, reducing savings (and raising dividends) or increasing borrowing to the point that the sum of cash reserves plus the unused credit line is not enough for the manager to deviate profitably. Consider now the option of becoming delinquent, represented by (4). The dividend paid is zero. Absent the agency problem, if cash in hand (now net of trade-credit purchases) is negative then positive borrowing covers bank-debt obligations with no residual left to trade-credit recovery; otherwise, the positive cash in hand is seized by trade-credit suppliers and no borrowing nor savings occurs. The introduction of the agency problem, however, may motivate increasing the level of borrowing, and therefore

[12] Strictly speaking, the firm is indifferent, and any arbitrarily small cost of borrowing will render this operation suboptimal. This fact is immediate in the absence of default risk as the firm's discount rate ρ coincides with risk-free discount price of debt Q. The result carries over to the present case with default risk as the net current gain to borrowing and failing to repay in some states is, under full information, offset by the risk-based pricing of debt.

trade-credit recovery, in order to eliminate the manager's incentive to divert. Appendix B.3 presents more formal details using the notation in eq (2) and (4).

Liquidation - Liquidation and exit means declaring bankruptcy if in debt, but is also open to firms without debts who yet cannot meet the fixed cost and cash input payments. There are two possible situations. The first is when cash-in-hand, $(1-\nu)ze^{\epsilon}F(x)-c_F-(1-\tau)px+\mathbf{I}_{b<0}(-b)$, is non-negative, in which case the value of the firm is zero and the residual is recovered by debt creditors as r^b.[13] The second case is when cash-in-hand is negative, meaning that the firm's output (plus possible reserves) would not cover the fixed cost and the cost of cash inputs, and the firm must therefore cease production altogether, with no recovery, while the firm must still cover the incurred cost of the cash inputs. We represent the firm's failure to produce by the indicator $d^f(\epsilon,b,\nu,x|S)=1$. When, otherwise, cash in hand is positive then $d^f(\epsilon,b,\nu,x|S)=0$. In sum,

$$\begin{aligned}
V^b(\epsilon,b,\nu,x|S) &= \begin{cases} 0 & \text{if } m(\epsilon,\nu,x|S)\geq 0 \\ -(1-\tau)p(S)x+\mathbf{I}_{b<0}(-b) & \text{otherwise} \end{cases} \\
d^f(\epsilon,b,\nu,x|S) &= \begin{cases} 0 & \text{if } m(\epsilon,\nu,x|S)\geq 0 \\ 1 & \text{otherwise} \end{cases} \\
\text{s.t. } m(\epsilon,\nu,x|S) &\equiv (1-\nu)ze^{\epsilon}F(x)-c_F-(1-\tau)p(S)x+\mathbf{I}_{b<0}(-b)
\end{aligned} \tag{5}$$

This results in the residual value $r^b(\epsilon,b,\nu,x|S)=\max\{(1-\nu)ze^{\epsilon}F(x)-c_F-(1-\tau)p(S)x,0\}$, for $b>0$, being recovered by bank-debt creditors. Dividends in this case π^b coincide with the value of the firm and can therefore be negative when the firm fails to operate in the period of liquidation:

$$\pi^b(\epsilon,b,\nu,x|S)=V^b(\epsilon,b,\nu,x|S) \tag{6}$$

Choice of repayment option - The optimal choice among the three options in the second stage gives decision rules $d^x(\epsilon,b,\nu,x|S)$ and $d^b(\epsilon,b,\nu,x|S)$ as the solution to

$$V(\epsilon,b,\nu,x|S)=\max\Big\{V^{ND}(\epsilon,b,\nu,x|S),V^x(\epsilon,b,\nu,x|S),V^b(\epsilon,b,\nu,x|S)\Big\}. \tag{7}$$

The demand for inputs - We now turn to the first stage within the period, before the realisation of the shocks. Taking V as given by (7), there the optimal choice yields the decision rule $x=x(\epsilon_{-1},b,\nu|S)$ that solves

$$W(\epsilon_{-1},b,\nu|S)=\max_x\sum_{\epsilon}\psi_{\epsilon}(\epsilon|\epsilon_{-1})V(\epsilon,b,\nu,x|S). \tag{8}$$

3.2.3 Entry, exit and distribution

The value of a new entrant $W^E(S)$ is the expectation of $W(\cdot)$ over the unconditional distribution on the starting ϵ_{-1} and b, μ^E, reflecting the assumption of zero initial debt financ-

[13] We are only describing situations where cash in hand is less than outstanding $b>0$. This simplifies notational burden as there is no recovery by trade credit in this case, an implication of the assumed seniority of bank debt. This focus is at no loss because if, otherwise, cash in hand exceeds $b>0$ then the option of liquidating

ing.[14] See details in appendix B.4. The free-entry condition is

$$W^E(S) \leq \xi^E, \tag{9}$$

with strict inequality only when there is zero entry, $m(S) = 0$.

The probability measure μ is defined over the ex-ante firm types (ϵ_{-1}, b, ν). We define the transition probabilities for existing firms $\text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S)$, where $\mathcal{B}'$ is a set containing elements b', and for entrants, for whom $\nu = 0$, $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S)$. These transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag, and will result in the transition function $H^\mu(\epsilon, \mathcal{B}', \nu' \mid S)$. The motion for the mass of existing firms N counts in the mass of current firms surviving into next period. For calculating outcomes affected by new firms, it will be convenient to define the post-entry probability distribution over firm's types as $\hat{\mu}(\epsilon_{-1}, \mathcal{B}, \nu)$. It accounts for the proportion $m(S)$ of new firms entering the market relative to the mass of firms N, as well as the firms in the probability measure μ already existing at the start of the period. For details about the distribution, see appendix B.5.

3.2.4 Trade-credit loss rate

Producers of intermediate inputs take as given the expected aggregate default rate on trade credit, or lost fraction of sales on trade credit, θ given the initial state S. This trade-credit loss rate results from aggregating up the individual firm's delinquency decisions $d^x(\epsilon, b, \nu, x|S)$ and bankruptcy decisions $d^b(\epsilon, b, \nu, x|S)$, given that their choice of inputs is determined by $x = x(\epsilon_{-1}, b, \nu|S)$. The loss rate also depends on the recovery from the delinquent firms' cash in hand left after repaying debts and cash inputs, which we have defined as $r^x(\epsilon, b, x|S, z)$. Specifically,

$$\theta(S) = \frac{\int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \Big(d^x(\cdot)(\tau p(S) x(\cdot) - r^x(\cdot)) + d^b(\cdot) \tau p(S) x(\cdot) \Big) \hat{\mu}(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)}{\int \sum_\epsilon \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \tau p(S) x(\cdot) \hat{\mu}(\mathrm{d}\epsilon_{-1} \times \mathrm{d}b \times \mathrm{d}\nu)} \tag{10}$$

where, for convenience, we are using the shorthand notation $r^x(\cdot) \equiv r^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^x(\cdot) \equiv d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, $d^b(\cdot) \equiv d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$, and $x(\cdot) \equiv x(\epsilon_{-1}, b, \nu|S)$, and the post-entry measure $\hat{\mu}$.

3.2.5 Remaining equilibrium conditions

Lenders and intermediation - Lenders use firm's decision rules and shock transition probabilities to infer the probability of debt default. They also take into account the recovery of the residual value of the firm. The price of debt can be written $q^{ND}(b', \epsilon, \nu|S) = Q(1 - \Lambda^{ND}(b', \epsilon, \nu|S))$ when there is no delinquency today, and $q^x(b', \epsilon|S) = Q(1 - \Lambda^x(b', \epsilon|S'))$

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with some trade credit recovery will be strictly dominated by delinquency and will never materialise.

[14] Ottonello and Winberry (2020), for instance, also make this assumption. The fraction of the entry cost

when there is delinquency, where $S' = H(S)$, and $\Lambda^{ND}(\cdot)$ and $\Lambda^{x}(\cdot)$ denote the corresponding forecasts of default losses, or expected default, which depend on the default rules $d^b(\cdot)$ and the recovery $r^b(\cdot)$ expressed as a rate over debt due b'. Regarding liquid reserves, to repeat, the spread in intermediation determines the discount price for cash savings $Q_0 = Qq_{\text{spr}}$, Explicit expressions are in appendix B.6.

Households - At the beginning of a period, the state for the representative consumer is (a, S), where $S = (N \times \mu, A, D)$, and a is the individual's risk-free asset.[15] The savings decision $a'(a \mid S)$, consumption $c(a \mid S, z)$, and labour supply $l(a \mid S)$ solve a typical optimisation problem. The aggregate dividend received by the household $\Pi(S)$ is determined by the firms' dividend policies from (3) and (6) (see appendix B.7). As standard, the first-order condition for the savings decision implies $u_c(c, l)Q = \beta u_c(c(a' \mid S'), l(a' \mid S'))$, and for labour supply $u_c(c(a \mid S), l)w(S) + u_l(c(a \mid S), l) = 0$. Recall that firms discount future values expected before the realisation of future shocks, and the appropriate rate is given by the household's discount factor based on a risk-free portfolio so $\rho = Q$. Details are in appendix B.8.

Aggregation and market clearing - Aggregate consistency requires individual assets a coincide with the aggregate A, and the evolution for the aggregate portfolio A' and the aggregate labour supply $L(S)$ be consistent with the household's policy functions and the distribution of firms. The evolution of external debt D' reflects the country's current account. Details are in appendix B.9.

3.3 A look into the mechanism

Before the dynamics, consider first the determination of the stationary equilibrium (see Appendix C.2 for its definition). Note that, with positive liquidation, in this stationary equilibrium it must be the case that entry is strictly positive so $m > 0$. Consistent with the algorithm outlined (see appendix C.3), the input price p mainly adjusts to satisfy the zero-profit entry condition (9), since p determines the profitability of final-goods firms. The flow rate of entrants m helps meet labour market clearing (28) as the mass of firms affects the aggregate level of demand for intermediate inputs x. The wage rate w meets the pricing condition of input producers (1). Trade-credit default θ, determined by firms payment decisions as in (10), therefore impacts the wage-to-input-price markup p/w, calling for lower wage or more expensive inputs.

The focus of this paper is actually on the contribution of trade-credit default to the transmission of fluctuations outside the steady state. Trade credit default brings two main distinctive mechanisms: the first one is the input price markup adjustment in general equilibrium; the second is the insurance that delinquency choice provides at the individual-firm level. The idea of the markup channel stems from the fact that an adverse shock can have

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financed by debt could be made positive.

[15] Even with aggregate shocks, contingent securities play no role here and we can think of a single bond. In Arellano, Bai, and Kehoe (2019) contingent securities are used in the context of a small open economy to provide full consumption insurance.

a direct impact on the estimated loss to trade credit, represented by θ. According to (10), this will come through the liquidation and delinquency policy rules, and the distribution of firms types. The rise in θ calls for an increase in the mark up w/p via the equilibrium pricing condition of input producers in Eq. (1). This adjustment can in principle happen through different combinations increases in the input price p or reductions in the wage w. The former means that final-goods firms face more expensive inputs and will respond by reducing their demand for inputs x, downsizing their production plans; the latter would reduce the supply of labour supply and of inputs, calling for reduced entry of firms. Either way, this will have detrimental consequences for GDP and employment.[16]

The insurance channel comes about because delinquency, one option in (7), may provide the firm the opportunity to avoid liquidation following an adverse shock. The contraction in the firm's demand for inputs, the decision represented in (8), will be less severe as a result, compared with the scenario where delinquency is not available, thus dampening the recession. The idea here is that under a bad aggregate shock the firm will be betting for survival on a narrower and thus higher-productivity set of idiosyncratic states, hence biasing the optimal decision towards more inputs, and the more so when delinquency provides downside risk insurance. On the other hand, while a given firm's specific type is in principle less likely to liquidate and exit when delinquency is available, firms also build lower precautionary buffers and are thus more prone to liquidation and exit. The sign of the consequences of the insurance channel for GDP is therefore ambiguous and will reflect the relative strength of, on one hand, its dampening effect by smoothing input demand and, on the other hand, its possible amplification effect by accelerating exit and slowing entry.

4 Quantitative benchmark model

In this section we consider the stationary equilibrium under specific functional forms and parameter values. Numerical values for the parameters will be chosen so that the model delivers realistic levels of aggregate trade credit and default, among other observable target variables. We will then consider properties of this model in terms of targeted and some non-targeted aggregate variables. The distribution and other equilibrium functions will be documented in Appendix D.

4.1 Specification

We specify the technology for final-goods firms as the concave production function

$$F(x) = x^{\gamma}, \text{ with } \gamma \leq 1.$$

The discrete Markov chain for the log of idiosyncratic productivity ϵ is chosen to approx-

[16]This model also contains the elements for a feedback effect whereby increased trade-credit default leads into further defaults and their resulting macroeconomic effects. Specifically, more expensive production inputs following the rise in trade-credit losses θ may be conducive to more bankruptcies and delinquencies, which will in turn be reflected in tighter credit conditions and costlier inputs, and increased trade-credit losses, and so forth.

imate a continuous first-order autoregressive process with persistence ρ_ϵ and where the innovations η follow an iid Normal distribution with standard deviation σ_η, of the form

$$\epsilon' = \rho_\epsilon \epsilon + \eta', \text{ with } \eta' \sim N(0, \sigma_\eta).$$

We choose a Markov chain with N_ϵ states and do the approximation following the discretization method in Tauchen (1986).

We assume an additively separable period utility function of the form

$$u(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - B\frac{l^{1+\phi}}{1+\phi},$$

where B is the weight on the disutility of work, $1/\phi$ is the Frisch elasticity of labour supply, and σ is the inverse of the intertemporal elasticity of substitution.

4.2 Calibration

A model's period corresponds to one year. In the stationary equilibrium, aggregate productivity is a constant parameter that can be normalised to $z = 1$. We can set the external debt and choose $D = 0$ so that this baseline is equivalent to a closed economy.

Assigned parameters - Several other parameters are assigned from direct observations. The parameters set directly are summarised in Table 2. The discount rate β is in this equilibrium equal to discount price Q and both are thus chosen to be equivalent to an annual rate of return of 4%. The interest wedge q_{spr} on liquid reserves implies a spread of about 2.5%, which seems realistic, even conservative.[17] The labour utility parameter ϕ corresponds to a Frisch elasticity of 2, and the intertemporal substitution σ is given a standard value. The curvature of the production function for final goods γ corresponds approximately to the labour share since we think of capital as given and uniform across firms, and we pick a value common in the literature, for instance Corbae and D'Erasmo (2021), Khan and Thomas (2013) or Arellano, Bai, and Kehoe (2019), which will also deliver the conventional labour share in Cooley, Prescott et al. (1995). The parameters of the idiosyncratic productivity process, ρ_ϵ and σ_η, are annual estimates from Compustat panel data on operating income obtained by Corbae and D'Erasmo (2021) which, as they indicate, are in line with estimates on quarterly data in the literature (e.g., Khan and Thomas (2013), Arellano, Bai, and Kehoe (2019), or Cooper and Haltiwanger (2006)). For the discrete approximation, we choose a number of states N_ϵ of 61.[18]

Internally calibrated parameters - The remaining seven parameters will be chosen so the

[17] See, e.g., Federal Reserve time-series evidence of short-term bills.

[18] Furthermore, as explained in Appendix C.4, we will introduce type I extreme-value shocks to final-goods firms' repayment choices and input hiring decisions that follow Gumbel distributions with a common dispersion parameter $\sigma_\zeta = 0.05$. This parameter's value, while helping convergence in computations, is of little consequence in that it will render delinquency and liquidation practically binary outcomes. See also Mateos-Planas, McCrary, Rios-Rull, and Wicht (2022).

Table 2: **Direct Parameters.**

parameter	value	observation
productivity	$z = 1$	normalisation
world discount price	$Q = 0.9615$	4% annual return
subjective discount rate	$\beta = 0.9615$	4% annual return
liquid return	$q_{\text{spr}} = 1.025$	2.5% spread
risk aversion	$\sigma = 2.0$	standard
utility labour	$\phi = 0.50$	Frisch elasticity 2.0
curvature final goods	$\gamma = 0.60$	60% approx labour share
persistence	$\rho_\epsilon = 0.653$	Corbae and D'Erasmo (2021)
volatility innovation	$\sigma_\eta = 0.20$	Corbae and D'Erasmo (2021)
number of productivity states	$N_\epsilon = 61$	

Note - Assigned parameters from direct observations and values used in the literature.

model matches a number of targets. These parameters are the proportion of intermediate input sales on trade credit τ, fixed cost c_F, the manager's outside productivity η,[19], penalty size for delinquency $\tilde{\nu}$, probability of forgiveness λ, cost of entry ξ^E, and utility weight of work B. Specifics about the procedure ca be found Appendix C.5.

Targets in the model - In this calibration, we will be targeting values for seven model's moments. The moments in the model are measures of firms' debt and operating income, bankruptcy, trade credit losses, the size of trade credit, and employment. For debt we use the average of the ratios of debt to operating income across firms, conditional on operating income being positive. For each, firm debt is given by the state variable b, and operating income is defined as revenues minus variable costs and fixed cost, that is $(1-\nu)ze^\epsilon F(x) - px - c_F$. The fraction of firms with strictly positive debt b gives the second debt moment. The fraction of firm with strictly positive operating income provides a third moment.

The bankruptcy rate in the model is the proportion of firms who liquidate when holding positive debt liabilities. It is calculated by integrating the bankruptcy policy rule $d^b(\epsilon, b, \nu, x)$ over the distribution of firms ex-post $\mu^{\text{ex-post}}(\epsilon, b, \nu, x)$, that is after their choice of input x and realisation of the shock ϵ, for strictly positive $b > 0$. The trade credit loss is measured as the fraction of intermediate input trade-credit sales that fail to perceive payment, adjusted by the possible recovery in cases of delinquency without liquidation. This corresponds to the variable θ from eq. (10) in the model. As the target for the size of trade credit, trade credit to GDP is measured as the value of intermediate inputs sold on trade credit as a proportion of the value of final goods produced. Let $\bar{x}$ the total amount of inputs purchased by final-goods firms, obtained by integrating the input demand policy function $x(.)$ over the distribution of final-goods firms, and let $\bar{y}$ the value of final output, obtained as the integral of firms' output $(1-\nu)e^\epsilon F(x)$ over the ex-post distribution of operating firms. The target ratio of trade credit to GDP is therefore $\tau p\bar{x}/\bar{y}$. Finally, labour supply is given

[19]The manager's salary w_m and η cannot be identified separately. All that matters is η/w_m. To reduce notation burden, we fix $w_m = 0.25$, and take η as the parameter to calibrate. We can always renormalise so that, as required, $w_m \leq c_F$.

by the household's optimal decision l.

Empirical targets - The empirical counterparts to the above targeted moments are derived from aggregate and firm-level data. For debt and operating income, we use firm-level data from Compustat for the period 1980-2014. Details are in Appendix A.3. We measure debt in the data as net of savings. This is the correct notion since it accommodates the model's endogenous state representing the (negative of the) net financial position. Savings is measured in the data as liquid assets since, with the model's focus on precautionary motives, we do not consider the decision on fixed capital or long-term investments. As for debt, for the main results, we use a measure of short-term debt liabilities, which we believe accords best with the scope of the model.[20] The resulting targets to match are a -0.61 average ratio of net debt to (positive) operating income, a 0.35 fraction of firms with positive net debt, and a 0.75 fraction of firms with positive income.

For the empirical counterpart to the trade-credit loss rate in the model, in Sec. 2 above, based on Compustat 1980-2016, we found the ratio of receivables doubtful over the sum of all receivables, doubtful or not, which motivates an empirical target for the loss rate on trade credit of 7%. For the size of trade credit, based of aggregate time series from the Fed and BEA on receivables and GDP, in Section 2 we constructed the ratio of trade credit to GDP which informs a 0.18 target ratio.

We choose the target for the bankruptcy rate based on the related literature. Corbae and D'Erasmo (2021) consider a rate close to 1% based on Compustat, and also indicate a 2 per cent based of their measure of distance to default. On the other hand, Ottonello and Winberry (2020) report a 3% default rate based on business survey data. All in all, we take the mid-value target of 2%. Finally, the target for employment is unity, a normalisation.

Table 3: **Calibrated Parameters**

parameter	calibrated value	target variables
fixed cost c_F	0.760	fraction in bankruptcy
side project η	0.078	debt to operating inc
penalty size $\tilde{\nu}$	0.153	fraction in debt
penalty forgiveness λ	0.332	fraction with positive operating inc
entry cost ξ^E	9.028	trade-credit loss rate
input sales on trade credit τ	0.290	trade credit to GDP
weight of work in utility B	2.458	labour supply

Note - Values of the seven internally calibrated parameters, and list of the targeted variables.

Calibration results - The values of the parameters are obtained from a procedure of minimisation of the sum of squared deviations between model and targets.[21] The calibrated

[20]Previous literature using models with a similar corporate balance-sheet structure have also measured debt as total, that is including long-term debt. Therefore, as a robustness exercise, we also carry out the analysis of a model calibrated to total debt. The implications for the transmission of shocks are comparable to the baseline.

[21]The algorithm is based on the Software BOBYQA, authored by M. J. D. Powell, to minimize sum of squares with bound constraints by combing trust region method and Levenberg-Marquardt method.

parameters values and targeted variables are summarised in Table 3.

This calibration implies that about 29% of sales of input are on trade credit. The delinquency penalty parameters imply for the firm a future loss of about 15% of output over, on average, a period of 3 years. The total operating fixed costs incurred by final-goods firms amounts to about 21% of the value of aggregate final output, above but not far from the replacement cost of capital of 16% implied by the capital-to-output ratio of 2.3 and depreciation rate of 6.9% in, for instance, Khan, Senga, and Thomas (2016).

Table 4 presents the results from the above parameters. The second column contains the implications of the model for the target moments. The match to the data is quite close, especially considering the highly parsimonious model and the complex interactions that complicate identification.[22]

Table 4: **Moments in model and data**

Moments	Data	Model
fraction in bankruptcy	0.02	0.02
(net) debt to operating inc	-0.61	-0.61
fraction in debt	0.35	0.35
fraction with positive operating inc	0.75	0.74
trade-credit loss rate	0.07	0.07
trade credit to GDP	0.18	0.17

Note - Values of targeted variables in the data and the calibrated model. Trade credit to GDP from Fed and BEA 1980-2016. Debt, operating income and trade-credit loss rate from Compustat 1980-2016. See Appendices A.1, A.2 and A.3. Author's calculations. Bankruptcy is based on Corbae and D'Erasmo (2021) and Ottonello and Winberry (2020).

For some sense of external validity, about 47% of firms have strictly positive dividends in the model which compares with about 51% in Compustat data. The endogenous exit rate is about 2.72%, which sits in the range between the exit rate of 1.20% in Corbae and D'Erasmo (2021) and the 5.2% in Khan, Senga, and Thomas (2016).[23] The model implies a labour share in GDP of 0.58, broadly in line with global and US measures by the 2010's (see Karabarbounis and Neiman (2014) and Ilo (2015)), thus supporting the initial choice of γ. Appendix D reports some policy functions and the distribution associated with this equilibrium.

5 Shocks and the markup channel

We have already discussed in Section 3.3 that this model contains the markup as one mechanism whereby trade-credit default will affect the determination of employment and output. The objective of this section is to study the role of this mechanism in the transmission of aggregate shocks, and also illustrate the dynamics within the model. We will

[22] Labour supply, not shown, matches its normalisation target exactly.

[23] The latter paper targets a total 10% exit rate, but 4.8% consists of exogenous departures.

find the general-equilibrium pricing effects associated with the occurrence of default are significant.

Since we do not model aggregate uncertainty, the focus here will be on unanticipated exogenous deterministic changes in the path of various parameters of the model, in the spirit of the so called MIT shocks (e.g., Boppart, Krusell, and Mitman (2018), Guerrieri and Lorenzoni (2017)). The description of the model needs to be extended slightly so that all the endogenous value and policy functions, and scalars, are suitably indexed by time.[24] One must also account explicitly for the distinct possibility of strictly zero entry in periods of contraction.[25] Since we consider a small open economy, along the adjustment path the price of bonds is constant and the market clearing condition in consumption may require external borrowing/saving during the transition, similarly to Arellano, Bai, and Kehoe (2019).[26] The level of consumption and external debt will therefore respond to reflect the wealth effect of the temporary shocks. Further details are in Appendix E.

We will study different aggregate shocks. The first one will consist of an increase in the fixed operating cost of firms c_F, which we think of as a financial shock. The second shock studied will be an increased volatility in the form of a rise in the standard deviation of idiosyncratic firm productivity σ_η. Finally, we will consider a reduction in total factor productivity z. We specify these shocks so that, while unanticipated, they become known to all types of firms right at the beginning of the impact period 0, before idiosyncratic shocks are realised, and thus will cause no losses or gains for input producers. We postulate these shocks follow an AR(1) adjustment process, being thus fully described by the size of the innovation at time zero and its persistence, and we can think of an impulse-response function (IRF) analysis. To discipline the exercise, we specify size and persistence to best approximate, following Arellano, Bai, and Kehoe (2019), the fall in the first year of about 4% and cumulative 9.2% fall during the 2 years of the Great Recession 2007-2009 in U.S. output. Appendix E contains the implementation details.

Table 5: **Endogenous markup and the transmission of shocks**

shock type	markup effect on GDP (%)	markup effect on employment (%)
financial c_F	41.7	442.4
volatility σ_η	25.3	47.1
tfp z	1.0	3.0

Note - For each shock, percentage of impact responses in GDP and employment that are due to the endogenous adjustment of the markup.

The paths that result from the shocks are not by and in themselves informative about the

[24]Equivalently, we could add those parameters in the description of S. The computation procedure follows the time-indexing option more literally.

[25]In contrast with the equilibrium logic when $m >> 0$, the free-entry condition (9) becomes a strict inequality when $m = 0$, so the wage will now have to fall to clear the labour market, and it is the price of inputs that will then adjust to meet the markup condition (1) rather than the entrants' zero-profit condition.

[26]Alternatively or in addition to considering an open economy, other restrictions like constant number of firms, are typically made in the literature studying dynamics, for instance Ottonello and Winberry (2020), Arellano,

role of the markup. In order to identify its role, we conduct an exercise where the markup channel of transmission of trade-credit default to the economy is switched off. Since, as we have learned already, it is the trade-credit loss rate θ that impacts the spread between the price of inputs and the wage cost, this exercise simply holds the spread at its stationary equilibrium level in the pricing equation (1). This amounts to asking, what if input producers failed to update the estimated payment risk they face? Table 5 summarises the main findings by displaying, for each shock, the percentage of the impact change of final output and of employment accounted for by the adjustment of the markup. These figures illustrate that generally the pricing of trade-credit default risk contributes to the size of output fluctuations to varying degrees across shocks, being substantial for the financial shock and the volatility shock, nearly 2/5 and 1/4 respectively, but much milder for the TFP shock. The influence of the markup on the response of employment is markedly larger in all cases. The rest of this section will analyse what lies behind these findings.

5.1 Financial shock

A financial shock consists of an increase in final-goods firms' fixed cost c_F. It is instructive to discuss first the response of aggregate quantities including output of final goods (GDP), employment and entry, and then the prices and default rates underpinning those changes. The quantities are shown by the solid lines in Figure 2 as proportional deviations from their stationary equilibrium values. On impact, the increased fixed cost of operation causes lower employment and output. Entry of firms m collapses to zero in the impact period and throughout the recession, and it is the reduction in the number of firms that explains the impact on output an employment.

Turning to prices, Figure 3, solid lines, displays the response of the input price, wage and measures of defaults. Trade-credit default increases by about 8.5 percentage points, driven by an increased frequency of both delinquencies and liquidations. The corresponding increase in the trade-credit loss rate, by the markup equation (1), drives a wider wedge between the price of inputs and the wage. The wage falls to clear the labour market on impact, and the price of the input increases to reflect that the markup, measuring trade credit default, rises. Both wage and price will be subsequently declining until output and entry resume.

Bai, and Kehoe (2019) and Bordalo, Gennaioli, Shleifer, and Terry (2021).

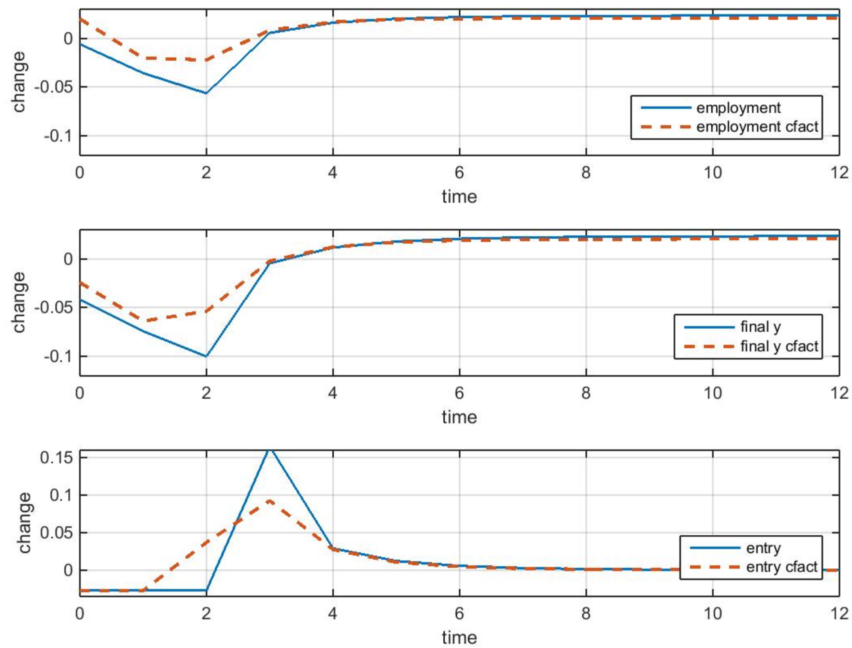

Figure 2: A financial shock. Quantities: employment (top), output (middle), entry (bottom). Baseline (solid lines) and constant markup (dashed lines).

What is the role of the markup? Regarding aggregate quantities, the dashed lines in Figure 2 show that there is a large markup effect on GDP and employment from trade-credit default, representing about 40 per cent of the response of output and more than the totality of the response of employment as seen earlier in Table 5. To understand what is behind these results, we turn to defaults and prices. As shown by the dashed lines in Figure 3, the constant-markup scenario implies a smaller fall in the wage and, accordingly, a smaller rise in the price of inputs.[27] Therefore, the rise in the markup has a downward effect on the wage and an upward effect on the input price. The former reduces the supply of labour, and the latter reduces the demand for inputs and therefore labour. Regarding default rates, the difference between the dashed lines and solid lines in Figure 3 indicates that there is some propagation of defaults (i.e., the changes in prices following from the surge in defaults induce in turn further delinquencies and liquidation of firms) although, as per the small differences in the graph, this propagation must be only a quantitatively small part of the total effects seen.

[27]The fall in the wage rate under the constant markup is in fact negative in this quantitative specification. The wage rising reflects that firms make a bet for survival by hiring more aggressively when faced with increased fixed costs of operation. This implies the impact increase of employment under the constant markup in Figure 2.

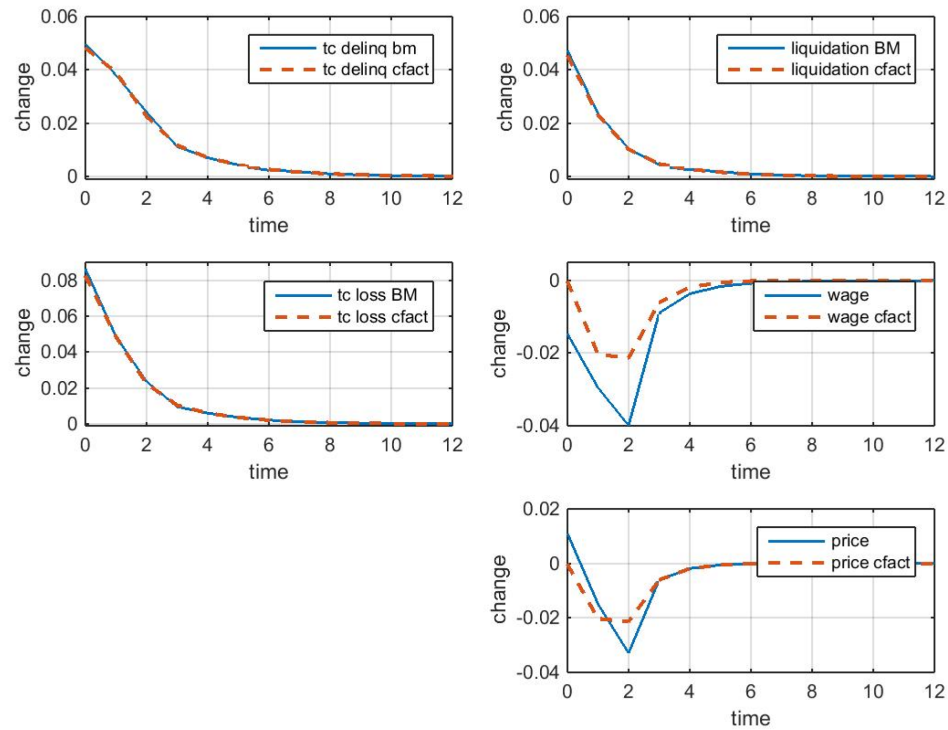

Figure 3: A financial shock. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (middle-left), wage w (middle-right), inout price p (bottom-right). Baseline (solid lines) and constant markup (dashed lines).

5.2 Volatility shock

Here we consider an increase in the standard deviation of the innovation to the process of individual firm's productivity, σ_η. The quantities are shown by the solid lines in Figure 4. The increased volatility causes lower employment and output and a collapse on entry to zero. This shock means that firms can expect wider swings going forward. Final-goods firms choose to reduce their demand for inputs. In this case, also entry of new firms falls to zero on the impact period. The response of the wage rate, input prices and defaults are displayed in Figure 5, by the solid lines. Trade-credit default increases as a result of more frequent delinquencies and bankruptcies, leading to a sharp rise in the trade-credit loss rate. (Default rates keep rising until entry of firms resumes.) The wage drops to clear the labour market initially on account of the reduced demand for inputs. The price of inputs also falls, but to a lesser extent than the wage does given that the intermediate producer must hedge via a higher markup against the increased losses from trade credit, i.e., Eq. (1).

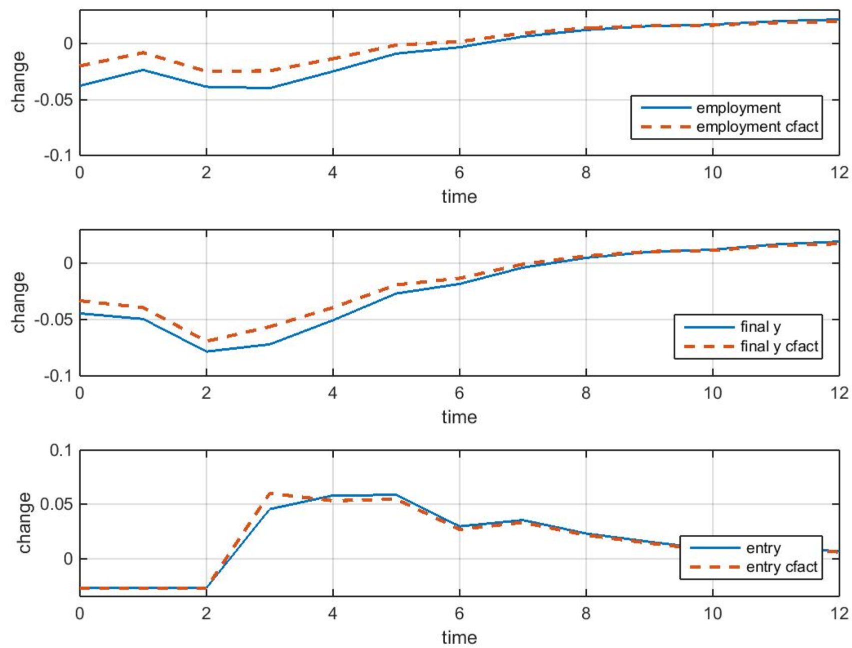

Figure 4: An increase in volatility. Quantities: employment (top), output (middle), entry (bottom). Baseline (solid lines) and constant markup (dashed lines).

There is thus indication that the markup must play a role, and to assess its quantitative importance we turn to the experiment. Consider Figure 4, dashed lines, corresponding to the constant-markup scenario. It depicts a contribution to GDP and employment from the markup adjustment that is substantial, representing 25% and 47%, respectively, of their responses as shown in Table 5 above. The prices and default measures under the constant markup, shown by the dashed lines in Figure 5, imply a markedly smaller fall in the wage and, accordingly, a larger fall in the price of inputs. Trade-credit default, via this price mechanism, exerts a downward effect on the wage and an upward effect on the input price thereby deterring workers' labour supply and firms' demand for inputs, and leading to lower employment and output. As shown in the panels displaying delinquency, liquidation and the trade credit loss, there is some feedback from trade-credit losses into further defaults which must make a small but non-negligible contribution to the overall effect of the markup channel.

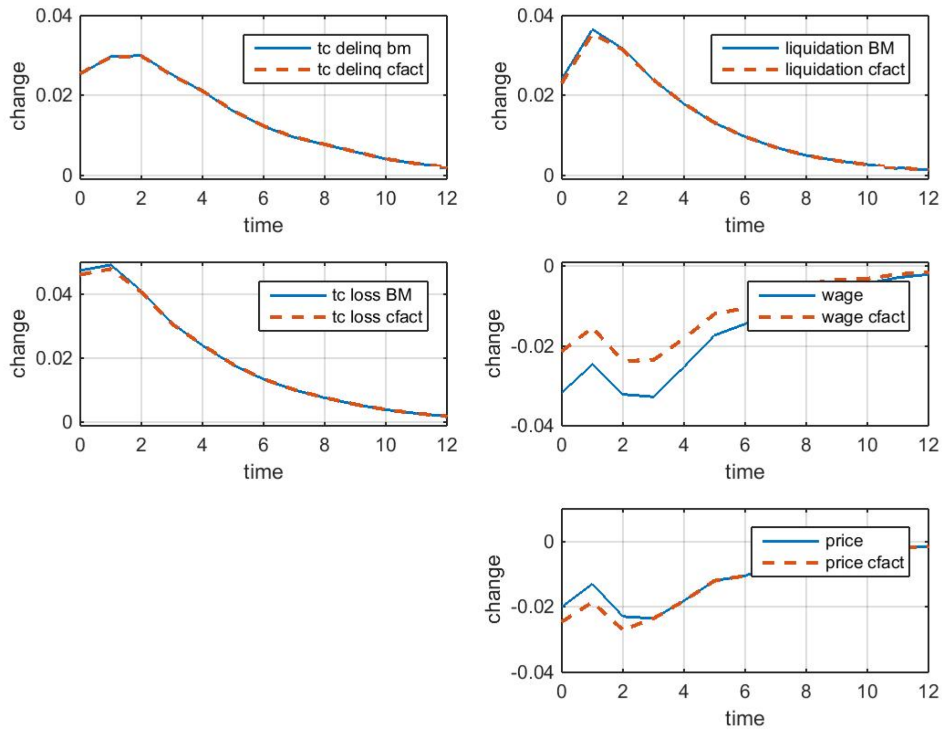

Figure 5: An increase in volatility. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (middle-left), wage w (middle-right), input price p (bottom-right). Baseline (solid lines) and constant markup (dashed lines).

5.3 Total factor productivity shock

The shock consists of a reduction in aggregate productivity z of final-goods firms in the impact period. The quantities are shown by the solid lines in Figure 6. The direct impact of the generalised fall in productivity is a reduction in the demand for inputs and production of existing firms, leading to lower employment and further reduced output. Entry of firms m collapses to zero on the impact and subsequent period, so there will be a net reduction in the number of firms over that subperiod. Figure 7, solid lines, displays the response of prices and default rates. Trade-credit default, from both delinquency and bankruptcy, increase mildly. The corresponding increase in the trade-credit loss rate will be driving a only slightly wider wedge between the price of inputs and the wage.

What if input producers fail to update their forecast of trade-credit default? Given the small shift in defaults, it will be of little consequence. Figure 6, dashed lines, shows the effect on employment and GDP coming from the markup effect, in the order of 1.0% as

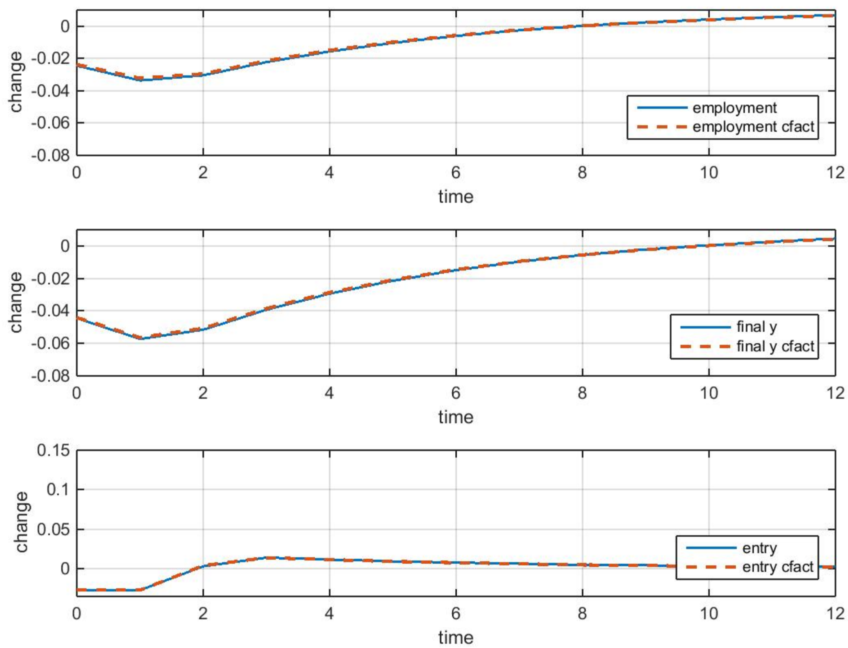

Figure 6: A fall in total factor productivity. Quantities: employment (top), output (middle), entry (bottom). Baseline (solid lines) and constant markup (dashed lines).

seen earlier in Table 5. Regarding prices, shown by the dashed lines in Figure 7, this constant-markup scenario implies only a slightly smaller fall in the wage and lower rise in default measures. The price mechanism of trade-credit default contributes very little to the transmission of the productivity shock.

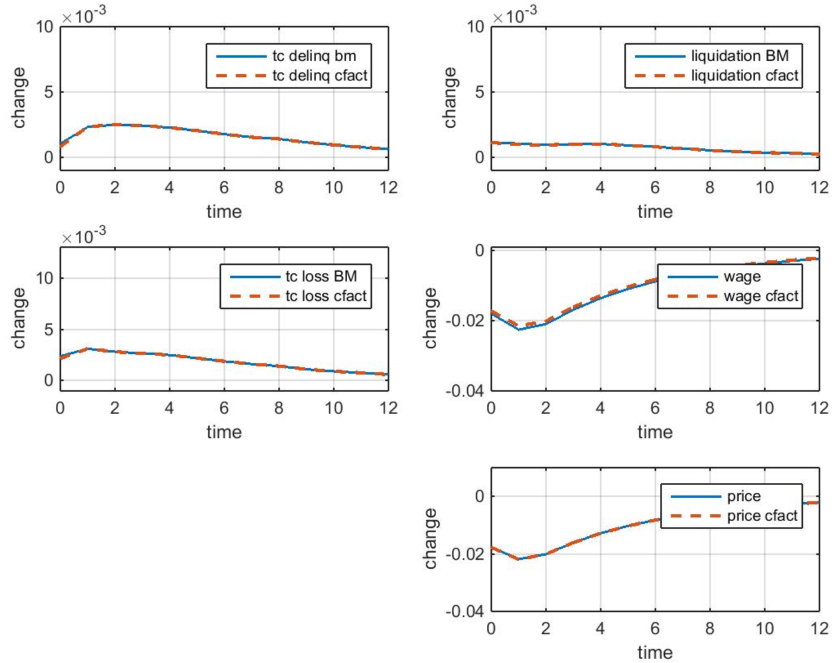

Figure 7: A fall in total factor productivity. Prices and default: delinquency rate (top-left), liquidation rate (top-right), trade-credit loss θ (middle-left), wage w (middle-right), input price (bottom-right). Baseline (solid lines) and constant markup (dashed lines).

6 Amplification and trade-credit default

How much and why does the consideration of trade-credit default affect the response of aggregate output to shocks? The preceding analysis suggests one reason why trade-credit default could amplify fluctuations, the markup channel as a representation of default spillovers. But this can only be part of the tale since in Section 3.3 we had identified another channel, insurance, and reasons why it might work towards either amplification or dampening. To investigate the matter, we consider the impact of shocks in an economy where there is no trade credit default. Given that we spare our model any non-essential frictions, this will be equivalent to an economy without trade credit.

We recalibrate the no-trade-credit counterfactual economy. A number of parameters become irrelevant, and the ones left to recalibrate internally are the manager's outside option η, the fixed costs c_F, the entry cost ξ^E, and the labour supply weight B. We pursue the same targets as in the baseline: bankruptcy, debt to operating income, fraction in debt, and the normalised labour supply. Table 6 displays the parameters and the targeted variables. Compared to the baseline in Tables 3 and 4, the counterfactual model features an increased manager's option value and a reduced fixed cost, which jointly work towards the targeted level of debt and bankruptcy rate. The lower fixed cost in the counterfactual balances out the lack of the insurance from trade-credit default that is instead present in the baseline model.

Table 6: **Calibration of zero trade-credit default counterfactual**

Parameters		Moments	
fixed cost c_F	0.55	debt to operating income	-0.62
entry cost ξ^E	13.77	fraction in bankruptcy	2.01
weight of work in utility B	2.06	fraction in debt	0.38
side project η	8.96	labor supply	1.00

Note - Model without tade-credit default. Parameters calibrated and model's implied moments.

To study the dynamic responses, we consider the exact same shocks as above in Section 5. We will break up the analysis of the amplification effect of trade-credit default on GDP into the amplification on its components: the size of population of firms given by entry m as $N(1+m)$, and their average output, the latter being represented by the combined firms' average demand for inputs x and their survival rate (i.e., 1 minus exit rate).[28]

The effects on the components of output will come through the two channels identified earlier: insurance (i.e., from the option to choose delinquency) and markup (i.e., from the endogenous general equilibrium price adjustment). Relative to the no-trade-credit counterfactual, the insurance mechanism of trade-credit default will mitigate the drop in the demand for inputs, a dampening effect, but may also reinforce the fall in survival (i.e., the rise in exit) and the fall in entry, an amplification effect. On its part, the markup mechanism, as seen in Section 5, makes for a sharper fall in the demand for inputs and further lowers survival, both having an amplification effect. The sign and size of GDP amplification from trade-credit default will therefore hinge on the strength of combined amplification forces (i.e., from the insurance channel on firms' survival and entry, and from the markup channel on both input demand and survival) relative to the strength of the dampening force coming from the insurance channel on input demand. Table 7 summarises the sign of the amplification role of these channels.

Table 8 presents, for each shock, the impact percentage responses of GDP and its components, and of the input price and wage rate, for the baseline and the counterfactual in the first and second rows respectively. The third row contains the implied amplification effect for each of the quantities. A consistent picture emerges where the input-demand dampening from the insurance channel is strong, and certainly dominates the input-demand amplification from the markup channel (hence, the negative signs at the bottom of the 4th column), but may or may not dominate the amplification of exit from the insurance channel (i.e., the positive sign at the bottom of the 5th column). Although, as seen in Section 5, it accounts for an important part of response to shocks, the markup general-

[28]Although we will be framing the analysis in terms of these averages, higher moments of the distribution of x's and survival rates must also play some part.

Table 7: **Sign of amplification effects on components of final output**

Channel	Entry	Input demand	Survival
Insurance	+ (entry down)	- (dem up)	+ (surv down)
Markup		+ (dem down)	+ (surv down)

Note - In parentheses, the direction of change in the corresponding variable underlying the sign of the amplification effect.

equilibrium mechanism (behind the contrasting prices in columns 6th and 7th across the two rows) plays a secondary role to the insurance mechanism regarding amplification from trade-credit default.

Table 8: **Amplification from trade-credit default**

	GDP	number of firms	average output	average input	survival fraction	p	w
Financial shock:							
Baseline	-4.10	-2.64	-1.46	2.12	-4.85	+1.11	-1.46
Counterfactual	-3.80	-2.31	-1.42	+0.13	-2.81	-2.18	-2.18
Amplification	9.76	14.27	2.56	-1549.04	72.39		
Volatility shock:							
Baseline	-4.34	-2.64	-1.75	-1.09	-2.48	-1.98	-3.12
Counterfactual	-6.36	-2.31	-4.14	-5.49	-1.89	-6.06	-6.06
Amplification	-31.65	14.27	-57.8	-80.02	30.86		
TFP shock:							
Baseline	-4.36	-2.64	-1.76	2.31	-0.12	-1.75	-1.78
Counterfactual	-4.02	-2.31	-1.74	0.13	-0.001	-1.77	-1.77
Amplification	8.48	14.27	0.95	-81.43	971.54		

Note - For each shock, impact percentage on GDP, its components, and input price and wage rate, for the baseline and counterfactual (no trade credit) economies, and the amplification effect in each of the components.

Financial shock - Consider first the financial shock in the top section of Table 8. Trade credit default has an amplification effect on output of about 10%. This is the result of a sharper fall in both the number of firms and their average output, compared to the counterfactual. The amplification from the number of firms of about 14% is driven by a sharper fall in entry in the trade-credit baseline. (Note this component will be identical across all shocks since all of them will imply a collapse of entry on impact.) The 3% amplification effect on average output reflects the effect of, compared to the counterfactual, a larger rise in exit in spite of a milder reduction in average input demand (an increase in this instance). With reference to the mechanisms outlined in Table 7 above, the input-demand dampening from the insurance channel overturns the input-demand amplification from the markup channel, but the survival amplification from the insurance channel is strong enough to produce net positive amplification overall.

Volatility shock - We now turn to the volatility shock reported in the middle section of Table 8. Trade credit default implies dampening, not amplification, of the output response, by over 30%. It is the result of a milder fall in average output, in spite of the sharper fall in the number of firms, compared to the counterfactual. The negative amplification of the average output reflects a much smaller reduction in average input demand despite the larger fall in survival. Again, in terms of the channels in Table 7 above, input-demand dampening from the insurance-channel strongly overturns the input-demand amplification from the markup channel, and the survival amplification from the insurance channel is too weak to offset it, resulting in net negative overall amplification.

TFP shock - Turning to the TFP shock, reported in the bottom section of Table 8, there is amplification of the output response of around 8%. It comes from both a sharper fall in number of firms and in average output. Average output mild amplification reflects the opposing effects of a larger fall in survival and a smaller fall in average input demand (i.e., a larger rise in this case). The input-demand dampening from the insurance channel outweighs the input-demand amplification from the markup channel, yet the survival amplification from the insurance channel is strong enough to produce positive yet weak amplification of average firm's output.

Reduced-form evidence? - We have conducted an exploratory empirical study of the amplification effects of trade credit using firm-level data. Details are in Appendix A.4, Table 9 and Table 10.[29] In firm fixed-effect regressions, using a firm's average trade credit in the interaction with industry growth, there is positive but statistically non-significant amplification when using sales growth, and statistically significant dampening when using employment growth. In industry fixed-effect regressions, there are no significant amplification coefficients, and the significance test for fixed effects fails. Our preceding quantitative analysis offers cues as to why these simple unconditional regressions may not be able to hint at the presence and sign of amplification from trade-credit. Further empirical analysis would benefit from an identification of the type of shocks affecting different periods and industries.

7 Concluding remarks

In this paper we ask whether trade-credit default may be quantitatively important for understanding fluctuations, and find that it may well be. The aggregate risk of failure in trade-credit payments is priced into the markup of supplied intermediate inputs, depressing wages or increasing the cost of inputs. On the other hand, the option of delinquency as insurance against liquidation smooths the contraction in firms' demand for inputs but

[29]We thank and follow here the very constructive comments of a referee.

can also lead to more exit and less entry. Our results arise from the interplay between these forces.

The model of heterogeneous firms presented here makes a meaningful contribution in its own right as it introduces the trade-credit default channel and therefore novel mechanisms for the role of the firm's financial position in aggregate fluctuations. Having made the case for the consideration of the trade-credit default in macroeconomic analysis, much work lies ahead in model extensions and their empirical validation.

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APPENDIX

A Appendix: Data

A.1 Aggregate data for evidence about trade credit

We describe the aggregate data used for Section 2.

Series retrieved from FRED Federal Reserve Economic Data, Economic Research Division, Federal Reserve Bank of St. Louis. Link: `https://fred.stlouisfed.org/series/`

Period and frequency: 1980-2016, quarterly.

Sources:

- Trade receivables: Board of Governors of the Federal Reserve System (US), Z.1 Financial accounts.
- GDP and deflator: U.S. Bureau of Economic Analysis (BEA)
- Charge-off rates: Board of Governors of the Federal Reserve System (US).

Variables:

- Trade receivables accounts:

  TRABSNNCB =Nonfinancial Corporate Business; Trade Receivables; Asset, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted

  TRABSNNB = Nonfinancial Noncorporate Business; Trade Receivables; Asset, Level, Billions of Dollars, Quarterly, Not Seasonally Adjusted

- Implicit GDP deflator:

  GDPDEF = Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted

- Real GDP:

  GDPC1 = Real Gross Domestic Product, Billions of Chained 2012 Dollars, Quarterly, Seasonally Adjusted Annual Rate

- Charge-off rates:

  CORBLACBS = Charge-Off Rate on Business Loans, All Commercial Banks, Percent, Quarterly, Seasonally Adjusted

Notes: The series for trade credit have been seasonally adjusted. GDP has been inflated to nominal to construct ratios of trade credit. The trade credit variables have been deflated to construct levels of time series. Cyclical variation is measured as the simple log difference of the time series.

A.2 Firm-level data for evidence about trade credit

We describe the firm-level data used for Sections 2 and Appendix A.4.

Source: Compustat. Accessed trough Wharton Research Data Service (WRDS) Compustat - Capital IQ, North America, Fundamentals Annual.

Period 1980-2016, annual frequency.

Exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000.

Variables:

- Trade credit: measured as *Accounts receivables - trade* (`rectr`);
- Trade-credit default: measured as *Receivable estimated doubtful* (`recd`).[30]
- Revenues or output: measured as *Sales* (`sale`).
- Employment: measured as *Employment* (`emp`)

Notes for aggregate analysis in Section 2: Trade credit loss rate must be constructed as `recd` over the sum `rectr+recd`. Construct time series as aggregates by year of receivables, sales, average trade credit loss level, and average trade credit loss rate, and then the implied ratios of trade credit to sales. For measuring time series properties of level variables, deflate aggregate receivables, sales, and trade credit loss, using the same GDP deflator as in sec A.1 above. Construct cyclical variation as log differences of level variables, and the difference for the loss rate.

A.3 Debt and operating income for calibration

In the calibration Section 4.2, for debt and operating income we use firm-level data from Compustat for the period 1980-2014. Accessed trough Wharton Research Data Service (WRDS) Compustat - Capital IQ, North America, Fundamentals Annual.

The raw data is cleaned in a way similar to Corbae and D'Erasmo (2021).We have borrowed some of their Stata code. We drop observations with 0 or missing assets, sales and property, plant and equipment, and exclude financial firms with SIC codes between 6000 and 6999, utility firms with SIC codes between 4900 and 4999, and firms with SIC codes greater than 9000. Observations are deleted if they do not have a positive book value of assets or if gross capital stock or sales are zero, negative, or missing.

Variables used for calibration as measured in Compustat:

- Operating income: measured as *Income before interest* (code `ebitda`).
- Short-term debt: measured as *Debt in current liabilities* (code `dlc`).

[30] Although the estimated doubtful accounts variable in Compustat includes both trade and non-trade accounts, trade accounts represent the vast majority, 90%, of all receivables (`rect`), and only about 2.5% of firms in the sample have any receivable accounts that include non-trade items.

- Long-term debt: measured as *Long-term debt* (code `dltt`).
- Liquid savings: measured as *Cash and short term investments* (code `che`), consisting of 'Cash Special Deposits, Working Funds and Temporary Cash Investments'.

Empirical target mean ratio of debt (or minus assets) to operating income: Observations with `ebidta` > 0. For net short-term debt of main text: mean `(dlc-che)/ebitda` with 2% trimming.

A.4 Regression analysis

In order to investigate the possibility of a systematic amplification role for trade credit we consider the variation contained in the firm-level micro data from Compustat described Appendix A.2. We study the question at both the firm and the industry levels.

Firm-level analysis - For the firm-level regression analysis we proceed as follows. Consider receivables, sales, and employment sorted by year, firms and industry. Construct trade credit ratio measured as receivables over sales. For sales growth regression: Deflate sales and receivables; regress firm sales growth against industry sales growth, an interaction of industry sales growth with firm's average trade credit ratio, and a firm's fixed effect. For employment growth regression: Deflate receivables; regress firm employment growth against industry employment growth, an interaction of industry employment growth with firm's average trade credit ratio, and a firm's fixed effect. Checked robustness to adding time fixed effects, and cleaning data by trade credit ratios.

We implement this idea first by thinking of fluctuations in terms of sales growth. We construct sales growth by firm and year, and total sales growth by industry and year. We then regress firm-level sales growth on industry-level sales growth, an interaction term of the industry-level growth with the firm-level average ratio of trade credit to sales, and a firm-level fixed effect. It is the coefficient of the interaction term that will be informative about amplification effects of trade credit. This interaction term consists of the product of the industry-year sales growth regressor and the firm average ratio of trade credit receivables to sales. We find a positive but not significant coefficient of the interaction term. This is shown on the first column of Table 9.

We also consider regressions where fluctuations are represented in terms of employment growth. We construct firm-year and industry-year employment growth as explained above. The estimated regression coefficient on the interaction term comes negative and significant. This is shown on the second column of Table 9.

Industry-level analysis - For the industry-level analysis, consider receivables, sales, and employment sorted by year, industry and in the aggregate. Construct trade credit ratio measured as receivables over sales. For sales growth regression: Deflate sales and receivables; regress industry sales growth against aggregate sales growth, an interaction of aggregate sales growth with industry's average trade credit ratio, and an industry fixed effect. For Employment growth regression: Deflate receivables; regress industry employment growth against aggregate employment growth, an interaction of aggregate employment growth with industry's average trade credit ratio, and an industry fixed effect. Checked robustness to time fixed effects and data trimming.

We investigate amplification by again thinking first of fluctuations in terms of sales growth.

Table 9: **Firm-level regressions**

Dependent variable	firm's sales growth	firm's employment growth
indust sales growth	0.2326	
	(0.00)	
indust sales growth × TC/sales	0.0295	
	(0.12)	
indust employment growth		0.1366
		(0.00)
indust employment growth × TC/sales		-0.0678
		(0.003)
number observations	175,023	163,090
R^2	0.1901	0.1671
Firm fixed effects	yes	yes
F-test fixed effects: prob>F	0.00	0.00

Note - Results from estimating versions of the specification:

$$\texttt{firm growth}_{jt} = \alpha_j + \beta\,\texttt{industry growth}_{jt} + \gamma\,\texttt{industry growth}_{jt} \times (\texttt{TC/sales})_j,$$

where j indexes the firm, t indexes period, α_j is a firm fixed effect, $\texttt{firm growth}_{jt}$ and $\texttt{industry growth}_{jt}$ corresponds to sales (1st column) or employment (column 2), in firm j and in firm j's industry respectively, and $\texttt{TC/sales}_j$ is firm j's average ratio of trade credit receivables to sales over the sample period. See Appendix A.2 for data sources and construction of variables. In parentheses, p-values. Adding time fixed effects do not practically affect the outcomes.

We construct sales growth by industry and year, and total aggregate sales growth by year, and, as explained, regress industry-level sales growth on aggregate-level sales growth, an interaction term of the aggregate-level growth with the industry-level average ratio of trade credit to sales, and an industry-level fixed effect. The interaction term consists of the product of the aggregate year sales growth regressor and the industry average ratio of trade credit receivables to sales. The amplification interaction coefficient is negative and not statistically significant. This is shown on the first column of Table 10.

We also consider regressions where fluctuations are represented in terms of employment growth. The estimated regression coefficient on the interaction term comes positive but borderline significant, and the test of significance of fixed effect fails amply. This is shown on the second column of Table 10.

B Appendix: Details of model

B.1 Price differentiation

In the model, the same price of the input p applies to both cash sales and trade-credit sales, describing the realistic situation where suppliers simply arrange the portion of any given sale to be on trade credit, and do not instead make some sales on trade credit and some other sales on cash with correspondingly different prices.

Specifying two different prices will not change our results though. Suppose prices for

Table 10: **Industry-level regressions**

Dependent variable	industry's sales growth	industry's employment growth
all sales growth	0.8828	
	(0.00)	
all sales growth × TC/sales	-0.0082	
	(0.67)	
all employment growth		1.0960
		(0.00)
all employment growth × TC/sales		0.0337
		(0.22)
number observations	12,681	12,599
R^2	0.0533	0.0391
Firm fixed effects	yes	yes
F-test fixed effects: prob>F	0.05	0.99

Note - Results from estimating versions of the specification:

$$\texttt{industry growth}_{jt} = \alpha_j + \beta\,\texttt{all growth}_t + \gamma\,\texttt{all growth}_t \times (\texttt{TC/sales})_j,$$

where j indexes the industry, t indexes period, α_j is an industry fixed effect, $\texttt{all growth}_t$ and $\texttt{all growth}_{jt}$ corresponds to sales (1st column) or employment (column 2) in the total sample, and $\texttt{TC/sales}_j$ is industry j's average ratio of trade credit receivables to sales over the sample period. See Appendix A.2 for data sources and construction of variables. In parentheses, p-values. Adding time fixed effects do not practically affect the outcomes.

cash sales and for trade-credit sales, p^C and p^T, with corresponding quantities x^C and x^T. Zero profit in each type of sale implies prices $p^C = w$ and $p^T = w/(1-\theta)$ and, over all, that $p^C x^C + (1-\theta)p^T x^T = wx$, or $(1-\tau)p^C + (1-\theta)\tau p^T = w$. Define the average price $p = (1-\tau)p^C + \tau p^T$, and note that it is an appropriate price index in that $px = p^C x^C + p^T x^T$. Writing the zero profit condition in terms of p yields $p - \theta\tau p^T = w$ which, using that $p^T = w/(1-\theta)$, implies the mark up $p/w = 1+\theta\tau/(1-\theta)$. This is slightly different from the markup we will obtain later but, if anything, it implies a larger elasticity and would strengthen the response of the markup to changes in θ, a main mechanism in the model.

B.2 The problem of the manager

- At night, the manager can use available funds $M > 0$ on a side project.
- They can purchase inputs x so $px = M$ for a payoff $f_m(\epsilon, x) = \eta(1-\nu)e^{\epsilon}F(x)$, with same specification as for the firm $F(x) = x^{\gamma}$.
- Maximisation over x gives profits the manager can extract from the side project: $\eta(1-\nu)e^{\epsilon}(M/p)^{\gamma}$.
- The outside value of the manager is a given exogenous w_m.
- Enforcement means the available funds M must satisfy $\eta(1-\nu)e^{\epsilon}(M/p)^{\gamma} \leq w_m$.
- This determines the implicit expression for the indifference threshold $M(\epsilon, \nu)$ in (2) and (4).

B.3 Final-goods firms characterisation

The description in Section 3.2.2 is based on the following analysis. Consider ND firms first, where the notation is as in (2). There are two cases.

- The first is when cih$(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) \leq 0$. In this case, $b' \leq 0$ and solves the savings problem $\max_{b' \in [b_L, 0]} \left\{ \text{cih}(\epsilon, \nu, b, x) + Q_0 b' + \rho \mathbb{W}'(\epsilon, \nu, b') \right\}$, with the bound on the level of savings $(-b_L)$ being the minimum of the cash in hand limit and the limit ensuring enforcement:

$$(-b_L) = \min \left\{ \frac{\text{cih}(\epsilon, \nu, b, x)}{Q_0}, \frac{-(B^*(\epsilon, \nu) - M(\epsilon, \nu))}{Q_0} \right\}.$$

- The second case is when, otherwise,
  - cih$(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) > 0$,
  - or cih$(\epsilon, \nu, b, x) < 0$.

  Here the solution has $b' > 0$ which is determined as follows:

  1. If cih < 0 and $-$cih $> B^*$: The option ND is not feasible so $V^{ND} = -\infty$.
  2. If, otherwise, cih ≥ 0 (and hence $B^* - M > 0$) or cih < 0 and $B^* - M > -$cih, then b' solves for the root of

  $$b' q^{ND}(b', \epsilon, \nu) - (B^* - M) = 0,$$

     implying possible positive dividends.
  3. Otherwise (i.e, if cih < 0 and $B^* - M < -$cih), zero dividends and b' solves:

  $$b' q^{ND}(b', \epsilon, \nu) - (-\text{cih}(\epsilon, \nu, b, x)) = 0.$$

Consider now the delinquent Nx firms, with the notation in (4). We distinguish two cases:

- The first is when cih$(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) \leq 0$. In this case, $b' = 0$ and recovery $r^x(\epsilon, \nu, b, x) =$ cih(ϵ, ν, b, x).
- The second case is when, otherwise,
  - cih$(\epsilon, \nu, b, x) \geq 0$ and $B^*(\epsilon, \nu) - M(\epsilon, \nu) > 0$,
  - or cih$(\epsilon, \nu, b, x) < 0$.

  Here the solution has $b' > 0$ determined as follows:

  1. If cih < 0 and $-$cih $> B^*$: option Nx is non feasible and $V^{Dx} = -\infty$
  2. If, otherwise, cih ≥ 0 (and hence $B^* - M > 0$) or cih < 0 and $B^* - M > -$cih, then b' solves

  $$b' q^x(b', \epsilon, \nu) - (B^* - M) = 0,$$

     with recovery $r^x = (B^* - M) - (-\text{cih})$.
  3. Otherwise (i.e, cih < 0 and $B^* - M < -$cih), b' solves

  $$b' q^x(b', \epsilon, \nu) - (-\text{cih}(\epsilon, \nu, b, x)) = 0,$$

     and yields zero recovery.

The discussion above assumes continuous debt price functions. Nonetheless, in this model the price of debt functions and therefore the resources raised via borrowing, may be discontinuous in the level of debt chosen. The reason is that at the level of debt where the firm may become delinquent with some positive probability at the end of the period, the marginal cost of hiring inputs drops and the firm's chosen amount of x may jump. One consequence is that the firm's choice of debt b' may raise excess resources and this residual would have to be apportioned to creditors accordingly. Our computational extension, as explained in Appendix C.4, will dispense with the existence of such discontinuities.

B.4 Entry

The value of a new entrant $W^E(S)$ is the expectation of $W(\cdot)$ over the unconditional distribution on the starting ϵ_{-1} and b, μ^E:

$$W^E(S) = \int W(\epsilon_{-1}, b, \nu = 0|S)\mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{11}$$

Here the probability distribution of entrants is

$$\mu^E(\epsilon_{-1}, b) = \begin{cases} \overline{\psi}_\epsilon(\epsilon_{-1}) & \text{for } b = 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

reflecting the assumption of zero initial debt financing.[31] The free-entry condition is

$$W^E(S) \leq \xi^E,$$

with strict inequality only when there is zero entry, $m(S) = 0$.

B.5 Distribution

The probability measure μ is defined over the ex-ante firm types (ϵ_{-1}, b, ν). It evolves according to $N' \times \mu' = H^\mu(S)$ where, as defined earlier, $S = (N \times \mu, A)$. We define the transition probabilities for existing firms $\text{Prob}(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b, \nu \mid S)$ and $\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S)$.. These transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag as shown in (17) and (18) below.

The motion for the mass of existing firms N counts in the mass of current firms surviving into next period, thereby

$$\begin{aligned} N' = N \times \int \sum_{\epsilon,\nu'} &\text{Prob}(\epsilon, \mathbb{R}, \nu'; \epsilon_{-1}, b, \nu \mid S)\mathrm{d}\mu(\epsilon_{-1}, b, \nu) \\ &+ Nm(S) \int \sum_{\epsilon,\nu'} \text{Prob}^E(\epsilon, \mathbb{R}, \nu'; \epsilon_{-1}, b \mid S)\mathrm{d}\mu^E(\epsilon_{-1}, b) \end{aligned} \tag{13}$$

[31] Ottonello and Winberry (2020), for instance, also make this assumption. The fraction of the entry cost financed by debt could be made positive. In this case, the debt issued to cover the fraction of the entry cost $\alpha^E\xi^E$ is

The transition function is

$$H^{\mu}(\epsilon,\mathcal{B}',\nu'\mid S)\equiv N\times\int \text{Prob}(\epsilon,\mathcal{B}',\nu';\epsilon_{-1},b,\nu,\mid S)\mathrm{d}\mu(\epsilon_{-1},b,\nu)$$
$$+\,Nm(S)\int \text{Prob}^{E}(\epsilon,\mathcal{B}',\nu';\epsilon_{-1},b\mid S)\mathrm{d}\mu^{E}(\epsilon_{-1},b),\quad (14)$$

where $\mu^E(\epsilon_{-1}, b)$ is the probability distribution of productivity and debt for new entrants. Now, since the distribution evolves as $N' \times \mu' = H^{\mu}(S)$, the probability measure follows

$$\mu' = H^{\mu}(S)/N'. \quad (15)$$

For calculating outcomes affected by new firms, it will be convenient to define the post-entry probability distribution over firm's types as $\hat{\mu}$. It accounts for the proportion $m(S)$ of new firms entering the market relative to the mass of firms N, as well as the firms in the probability measure μ already existing at the start of the period. Therefore $(N + Nm(S))\hat{\mu}$ is the total scaled up measure of firms. Given $m(S)$, N and μ and μ^E, the post-entry probability measure obtains as

$$\hat{\mu}(\epsilon_{-1},b,\nu)=\frac{\mu(\epsilon_{-1},b,\nu)\times N+\mu^{E}(\epsilon_{-1},b)\mathcal{I}_{\nu=0}\times Nm(S)}{N+Nm(S)}. \quad (16)$$

More formally, μ belongs in the set of probability measures over a measurable space consisting of the set of elements of the individual final-goods firm's state, and the product of the corresponding Borel algebras.

The transition probabilities are given by the firms' optimal decisions and the process for the delinquency flag. For existing firms

$$\text{Prob}(\epsilon,\mathcal{B}',\nu';\epsilon_{-1},b,\nu\mid S)=\begin{cases}\psi_{\epsilon}(\epsilon\mid\epsilon_{-1}) & \text{if}\;\; g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad\; d^{b}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; d^{x}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; \nu'=0,\;\nu=0\\ \lambda\psi_{\epsilon}(\epsilon\mid\epsilon_{-1}) & \text{if}\;\; g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad\; d^{b}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; d^{x}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; \nu'=0,\;\nu=\tilde{\nu}\\ (1-\lambda)\psi_{\epsilon}(\epsilon\mid\epsilon_{-1}) & \text{if}\;\; g^{ND}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad\; d^{b}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; d^{x}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; \nu'=\tilde{\nu},\;\nu=\tilde{\nu}\\ \psi_{\epsilon}(\epsilon\mid\epsilon_{-1}) & \text{if}\;\; g^{x}(\epsilon,b,x(\epsilon_{-1},b,\nu\mid S)\mid S)\in\mathcal{B}'\\ & \quad\; d^{b}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=0\\ & \quad\; d^{x}(\epsilon,b,\nu,x(\epsilon_{-1},b,\nu\mid S)\mid S)=1\\ & \quad\; \nu'=\tilde{\nu}\\ 0 & \quad\; \text{otherwise}\end{cases} \quad (17)$$

given by $b = b^E : q^E(b^E) = \alpha^E \xi^E$. Since the productivity type is unknown before entry, competitive lenders price this debt by pooling across types according to the function $q^E(b')$ such that $q^E(b) = \sum_{\epsilon_{-1}} \overline{\psi}_{\epsilon}(\epsilon_{-1}) q^{ND}(b, \epsilon_{-1}, \nu =$

For new entrants, for whom $\nu = 0$,

$$\text{Prob}^E(\epsilon, \mathcal{B}', \nu'; \epsilon_{-1}, b \mid S) = \begin{cases} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^{ND}(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) \in \mathcal{B}' \\ & \quad\;\; d^b(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\;\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\;\; \nu' = 0 \\ \psi_\epsilon(\epsilon \mid \epsilon_{-1}) & \text{if} \quad g^x(\epsilon, b, x(\epsilon_{-1}, b, 0 \mid S) \mid S) \in \mathcal{B}' \\ & \quad\;\; d^b(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 0 \\ & \quad\;\; d^x(\epsilon, b, 0, x(\epsilon_{-1}, b, 0 \mid S) \mid S) = 1 \\ & \quad\;\; \nu' = \tilde{\nu} \\ 0 & \quad\;\; \text{otherwise} \end{cases} \tag{18}$$

B.6 Lenders

The price of debt can be written

$$q^{ND}(b', \epsilon, \nu|S) = Q(1 - \Lambda^{ND}(b', \epsilon, \nu|S)) \tag{19}$$

when there is no delinquency today, and

$$q^x(b', \epsilon|S) = Q(1 - \Lambda^x(b', \epsilon|S')) \tag{20}$$

when there is delinquency, where where $S' = H(S)$, and $\Lambda^{ND}(\cdot)$ and $\Lambda^x(\cdot)$ denote the corresponding forecasts of default losses or expected default, which depend on the default rules $d^b(\cdot)$ and the recovery $r^b(\cdot)$ expressed as a rate over debt due b'.

Expressions for Λ^{ND} And Λ^x follow. For a firm not defaulting, let the lender's expected recovery rate

$$rec^b(\epsilon', b', \nu' \mid S') \equiv \frac{r^b(\epsilon', b', \nu', x(\epsilon, b', \nu'|S') \mid S')}{b'}.$$

Therefore the expected default $\Lambda^{ND}(\cdot)$ in Eq. (19) can be written

$$\begin{aligned} &\Lambda^{ND}(b', \epsilon, \nu|S') \equiv \\ &\qquad \mathcal{I}_{\nu>0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ (1-\lambda) d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) \\ &\qquad\qquad + \lambda d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\} \\ &\qquad\qquad\quad + \mathcal{I}_{\nu=0} \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', 0, x(\epsilon, b', 0|S')|S')(1 - rec^b(\epsilon', b', 0 \mid S')) \Big\} \end{aligned}$$

The expected default for a firm incurring delinquency in (20) is

$$\Lambda^x(b', \epsilon|S') = \sum_{\epsilon'} \psi_\epsilon(\epsilon'|\epsilon) \Big\{ d^b(\epsilon', b', \tilde{\nu}, x(\epsilon, b', \tilde{\nu}|S')|S')(1 - rec^b(\epsilon', b', \tilde{\nu} \mid S')) \Big\}$$

Regarding liquid reserves, to repeat, the spread in intermediation determines the discount price for cash savings:

$$Q_0 = Q q_{\text{spr}} \tag{21}$$

0). Here we have decided to keep $\alpha^E = 0$ for simplicity.

B.7 Aggregate dividends

The aggregate dividend received by the household/shareholder can be represented by the expression shown in (22) for $\Pi(S)$.

Given the firms' dividend policies from (3) and (6), the aggregate dividend is

$$\Pi(S) = \sum_{\epsilon} \psi_\epsilon(\epsilon \mid \epsilon_{-1}) \left[\int \mathcal{I}_{d^b(\dots)=0, d^x(\dots)=0} \pi^{ND}(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \right.$$
$$\left. + \int \mathcal{I}_{d^b(\dots)=1} \pi^d(\epsilon, b, x(\epsilon_{-1}, b, \nu \mid S) \mid S) \mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \right] (N + m(S)N)$$
$$- \xi^E m(S) N \quad (22)$$

where we are using the shorthand notation $d^x(\cdot) \equiv d^x(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$ and $d^b(\cdot) \equiv d^b(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S)|S)$.

B.8 Households

At the beginning of a period, the state for the representative consumer is (a, S), where $S = (N \times \mu, A, D)$, and a is the individual's risk-free asset.[32] The savings decision $a'(a \mid S)$, consumption $c(a \mid S, z)$, and labour supply $l(a \mid S)$ solve

$$U(a \mid S) = \max_{\{a', l\}} \{u(c, l) + \beta U(a' \mid S')\} \quad (23)$$

subject to

$$c + Qa' = w(S)l + a + \Pi(S),$$

where the components of S' obey $S' = H(S)$. As standard, the first-order condition for the savings decision implies $u_c(c, l)Q = \beta u_c(c(a' \mid S'), l(a' \mid S'))$, and for labour supply $u_c(c(a \mid S), l)w(S) + u_l(c(a \mid S), l) = 0$.

The firms' dividend policies from (3) and (6) determine the aggregate dividend received by the household/shareholder $\Pi(S)$. Details are in appendix B.7.

Recall firms discount future values expected before the realisation of future shocks. The appropriate rate is given by the stochastic discount factor based on a risk-free portfolio. From the consumption first-order condition, this means

$$\rho = Q \quad (24)$$

B.9 Market clearing

Aggregate consistency requires individual assets coincide with the aggregate:

$$a = A. \quad (25)$$

[32] Even with aggregate shocks, contingent securities play no role here and we can think of a single bond. In

Given the household's policy functions $a'(a \mid S, z)$ from (23), the transition function for the aggregate portfolio A' is

$$H^A(S) = a'(A \mid S). \tag{26}$$

Similarly, aggregate labour supply and consumption are given by

$$L(S) = l(A \mid S) \text{ and } C(S) = c(A \mid S). \tag{27}$$

Equilibrium requires clearing in the market for labour, final output and assets. By Walras' Law, we only need to consider the first two. Clearing in the labour market means[33]

$$L(S) = N \times \int x(\epsilon_{-1}, b, \nu, \mid S)\mathrm{d}\mu(\epsilon_{-1}, b, \nu) + Nm(S) \int x(\epsilon_{-1}, b, \nu = 0 \mid S)\mathrm{d}\mu^E(\epsilon_{-1}, b). \tag{28}$$

For final goods, the condition is

$$\begin{aligned} C(S) + \xi^E Nm(S) + D - Q\ H^D(S) &= (N + Nm(S)) \\ &\times \sum_{\epsilon} \left[\int \psi_\epsilon(\epsilon \mid \epsilon_{-1})((1-\nu)z\epsilon F(x(\epsilon_{-1}, b, \nu|S)) - c_F)(1 - d^b(\cdot)d^f(\cdot))\mathrm{d}\hat{\mu}(\epsilon_{-1}, b, \nu) \right] \end{aligned} \tag{29}$$

where $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ and $d^b(\cdot) \equiv d^d(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$. The two terms on the right correspond to existing and new entrants, respectively. The possibility of firm failure is captured by the failure indicator $d^f(\cdot) \equiv d^f(\epsilon, b, \nu, x(\epsilon_{-1}, b, \nu|S))$ which is 1 when the firm declares bankruptcy and cannot cover the fixed cost and payments for cash inputs.

C Appendix: Equilibrium

C.1 Definition of equilibrium

An equilibrium consists of the functions

- For final-goods firms: policy rules $\{g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, \pi^{ND}, \pi^b, x\}$, and value functions $\{V^{ND}, V^b, V^x, W\}$.
- Loan price functions: q^{ND}, q^x.
- Input price function p.
- Wage function w.
- Aggregate trade-credit default function θ.
- Aggregate dividends Π.
- Policy functions for households: a', l, c.
- Transition function for firms' distribution H^μ.
- Transition function for household assets H^A.
- Transition function for external debt H^D.

Arellano, Bai, and Kehoe (2019) contingent securities are used in the context of a small open economy to provide full consumption insurance.

[33] We could also write it in terms of $\hat{\mu}$, that is $l(S) = (N + Nm) \int x(.)\mathrm{d}\hat{\mu}(.)$, but the present renders more clearly the role of entry $m(S)$ in market clearing.

- Aggregate labour supply L.
- The probability distribution of entrants μ^E (and b^E and q^E).
- The measure μ and mass N of firms.
- Post-entry measure $\hat{\mu}$.
- Rate of entry m.
- Value of entry W^E.

and, for the given risk-free discount price Q, the scalars

- Discount rate for firms and lenders ρ.
- Cash discount price Q_0.

They must satisfy the following conditions:

1. Final-good firms: Given q^{ND}, q^x, ρ, p, and H, final-good firms's outcomes g^{ND}, g^x, r^b, r^x, d^b, d^f, d^x, π^{ND}, π^b, and x, solve (2), (3), (4), (6), (5), (7), (8).
2. Lenders zero profit: Given H, Q, and d^b, r^b, and x, debt prices q^{ND} and q^x satisfy equations (19) and (20).
3. Free-entry intermediation of liquid reserves: Given Q, Q_0 satisfies (21).
4. Distribution transition: Given g^{ND}, g^x, d^x, d^b and x, the transition probabilities Prob and Prob^E are determined by (17), (18). Given those and m, N, μ, μ^E, then H^μ and $N'\mu'$ follow (14), (13), and (15).
5. Post-entry distribution: Given m, N, μ, μ^E, then $\hat{\mu}$ is given by (16).
6. Distribution of entrants: Given q^{ND}, (12) determines $\mu^E(\epsilon_{-1}, b)$, and b^E q^E.
7. Labour market clearing: The functions m, μ, μ^E, N, L, and x satisfy (28).
8. Trade-credit loss and failure: Given d^x, r^x, x, d^b, d^f, p and $\hat{\mu}$, then θ is given by (10).
9. Input-producers: The functions p, w and θ satisfy (1).
10. Free entry: Given W and μ^E, the function W^E in (11) and p satisfy (9).
11. Discount: Given Q, ρ is determined by (24).
12. Aggregate dividends: Given d^b, d^x, x, π^{ND} and π^b, m, N and $\hat{\mu}$, then Π is given by (22)
13. Consumer: Given H, Q, w and Π, the household's $a'(\cdot), l, c$ solve the problem in (23).
14. Clearing in final goods. Given C, D, m, x, $\hat{\mu}$ and N, and d^b and d^f, eq. (29) holds.
15. Aggregate consistency: a', H^A, L and C satisfy (25), (26) and (27).

By Walras' law, we omit the condition of clearing in the market for securities.

C.2 Stationary equilibrium

A stationary equilibrium which is one where the aggregate state S is constant over time. Therefore, there the endogenous equilibrium functions will be constant functions, and prices and discount rates scalar numbers p, w, Q and ρ, and so will the quantities m, N, l, c, and θ. Note that, with positive liquidation, in this stationary equilibrium it must be the case that entry is strictly positive so $m > 0$.

The algorithm to find the stationary equilibrium exploits a form of block recursivity of this equilibrium. The input price p can be found iteratively as the one that solves the final-goods firm's free-entry condition by using the equilibrium functions for final-goods firms and lenders. Note that these limited set of conditions suffice to fully determine p because strictly positive entry m means that the zero-profit condition for final-goods firms binds. Given p, then a second block determines aggregates including the distribution, trade-credit loss, and the wage via the input producer's pricing conditions, as well as firm entry, the mass of firms, and labour supply to meet market clearing. Details are in Appendix C.3.

The computation of this model- particularly the joint determination of final-good firms' choices and lenders' debt pricing- presents some practical challenges related to the potential discontinuities already mentioned and to the inherent non-convexities introduced by the binary repayment outcomes. To overcome issues of accuracy and convergence in the iterations, for computational purposes we introduce type-I extreme-value shocks affecting the discrete choices of delinquency and liquidation, and also the level of a firm's demand for input. For good reason, this approach turns out to dispense with the complications noted. Appendix C.4 provides more details.

C.3 Algorithm for the stationary equilibrium

In a stationary equilibrium, the Euler equation associated with (23) requires $\beta = Q$ and then, by (24), we can pin down the equilibrium discount as

$$\rho = Q = \beta.$$

The labour supply optimality condition associated with (23) reduces to $u_c(c, l)w(S)+u_l(c, l) = 0$. This condition implies, via (27), aggregate labour supply L as a function of c and w which here we denote $l^S(c, w)$.

For a predetermined D, a stationary equilibrium can be constructed through the following steps:

1. Guess price p.
2. Final-goods firms and lenders: $p \to q, x, g, \pi, r, d, V, W$
   (a) Firms Eq. (2) to (8): $p, q \to x, g, \pi, r, d, V, W$.
   (b) Lenders Eq. (19) and (20): $d, r, x \to q$.
3. Distribution entrants: By (12), 3(i) $q \to q^E \to b^E$, and 3(ii) $b^E \to \mu^E$.
4. Final goods free entry: $W, \mu^E \to W^E$ by Eq. (11).
5. Update p via Eq. (9). Back to 2.
6. Distribution: $d, g, x, \mu^E \to \mu, m$
   (a) Guess μ.
   (b) Find m to match exit by Eq. (13).
   (c) Update μ by transition function Eq. (14), (17), (18), and (15). Back to 6b.
7. Post-entry distribution: $\mu, m, \mu^E \to \hat{\mu}$ by Eq. (16).
8. Trade credit loss: $\hat{\mu}, d, x, r, p \to \theta$ by Eq. (10).
9. Input pricing: $p, \theta, \eta \to w$ by Eq. (1).
10. Market clearing:
    (a) Guess N.
    (b) Consumption: $D, N, m, x, d, \hat{\mu} \to c$ by Eq. (29).

(c) Labour supply: c, w, $\rightarrow L$ from (23) as given by $l^S(.,.)$ above via (27).
(d) Labour clearing: $\mu^E, \mu, m, x, L \rightarrow N$ by Eq. (28). Back to (b).

Note the first 5 steps fully determine p iteratively. The remaining steps are direct, except for iterations in determining the distribution and market clearing.

C.4 Computation with extreme-value shocks

The main block in the characterisation of an equilibrium is the joint determination of final-goods firms choices and lenders loan pricing, for a given price of the intermediate input p. Firms take as given the loan pricing functions in (19) and (20). Lenders take as given the firm's default and delinquency policy rules resulting from (2) through to (8). We define all value functions and policy functions on grids for the states ϵ, b, ν, and x.

Repayment options. The liquidation, delinquency or repayment outcome is a discrete choice variable and we extend the model with extreme-value shocks affecting these choices. In the decision problem of the firm among the three options in (7), we introduce shocks ζ^{ND}, ζ^x and ζ^b, associated with the decisions of repaying, delinquency, and liquidating, respectively. They are collected in a vector ζ. The decision problem is subject to an extra exogenous state ζ and is transformed into

$$\tilde{V}(\epsilon, b, \nu, x, \zeta) = \max\left\{V^{ND}(\epsilon, b, \nu, x) + \zeta^{ND}, V^x(\epsilon, b, \nu, x) + \zeta^x, V^b(\epsilon, b, \nu, x) + \zeta^b\right\}, \qquad (30)$$

which gives decision rules $\tilde{d^x}(\epsilon, b, \nu, x, \zeta)$ and $\tilde{d^b}(\epsilon, b, \nu, x, \zeta)$. (I omit the aggregate states S for notational simplicity here.) Assume that these shocks follows a Gumbel, or Type-I extreme value, distribution with location and scale parameters μ_ζ and σ_ζ, that is $\zeta^j \sim G(\mu_\zeta, \sigma_\zeta)$, with cdf

$$F(\zeta^j) = \exp\left(-e^{-\frac{\zeta^j - \mu_\zeta}{\sigma_\zeta}}\right),$$

$$\text{mean}(\zeta^j) = \mu_\zeta + \sigma_\zeta\gamma, \text{ with } \gamma = 0.5772, \text{ var}(\zeta^j) = \sigma_\zeta^2\pi^2/6, \text{ mode}(\zeta^j) = \mu_\zeta.$$

Given this specification, the solution to this problem can be characterised in terms of a distribution of ex-ante probabilities among the different options, which are functions of the "fundamental" values V^j's. Specifically:

$$\begin{aligned} d^x(\epsilon, b, \nu, x) &= \frac{e^{V^x(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}} \\ d^b(\epsilon, b, \nu, x) &= \frac{e^{V^b(\epsilon,b,\nu,x)/\sigma_\zeta}}{\sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}} \end{aligned}$$

On the other hand, the expected value

$$V(\epsilon, b, \nu, x) \equiv \mathbb{E}\tilde{V}(\epsilon, b, \nu, x, \zeta) = \mu_\zeta + \sigma_\zeta\gamma + \sigma_\zeta \log \sum_{j\in\{ND,x,b\}} e^{V^j(\epsilon,b,\nu,x)/\sigma_\zeta}.$$

We normalise by a choice of μ_ζ so that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = 0$. It is known that $E\max\{\zeta^{ND}, \zeta^x, \zeta^b\} = \mu_\zeta + \sigma_\zeta\mu_\zeta + \sigma_\zeta \log J$ where $J = 3$ is the number of discrete options. Therefore our normalisation implies $\mu_\zeta = -\sigma_\zeta\mu_\zeta - \sigma_\zeta \log J$. Finally, to deal with issues of computer arithmetics in the evaluation of the exponential function, we perform the calculations under some convenient but innocuous transformations.

Demand for inputs. For a given discrete state, the decision about the quantity of input $x(\epsilon_{-1}, b, \nu)$ solving (8) can be computed on the discrete grid for x. In examples, we have found x to show discontinuities and unwarranted patterns. Although extreme-value shocks in repayment discrete choices help resolve some of these issues, we have also introduced shocks affecting the choice of x_i on the grid $X = \{x_1, ..., x_{N_x}\}$, given by ζ_i following a Gumbel distribution, which we stack in a vector ζ. The original problem in (8) is reformulated as

$$\tilde{W}(\epsilon_{-1}, b, \nu, \zeta) = \max_i \{RHS^x(x_i \mid \epsilon_{-1}, b, \nu) + \zeta_i\}_{i=1}^{N_x},$$

where we define

$$RHS^x(x_i \mid \epsilon_{-1}, b, \nu) \equiv \sum_{\epsilon} \psi_\epsilon(\epsilon|\epsilon_{-1}) V(\epsilon, b, \nu, x_i).$$

The solution can be described as an ex-ante probability distribution over the choices given by

$$\text{prob}^x(x \mid \epsilon_{-1}, b, \nu) = \frac{e^{RHS^x(x|\epsilon_{-1},b,\nu)/\sigma_\zeta}}{\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta}}.$$

The maximised value

$$W(\epsilon_{-1}, b, \nu) = E[\tilde{W}(\epsilon_{-1}, b, \nu, \zeta)] = \mu_\zeta + \sigma_\zeta \gamma + \sigma_\zeta \log \left[\sum_{x_i} e^{RHS^x(x_i|\epsilon_{-1},b,\nu)/\sigma_\zeta} \right].$$

Normalising so the expected max of the shocks is zero means $\mu_\zeta + \sigma_\zeta \gamma + \sigma_\zeta \log N_x = 0$.

Borrowing. The borrowing decision in (2) and (4) is computed as a continuous variable as in the theoretical characterisation provided in Appendix B.3. However, one consequence of the possible discontinuity in the price of debt is that the value of resources raised by borrowing will also have one discontinuity in b'. In some cases, it will not possible to find the exact zero, which is when the solution corresponds to the discontinuity point. In other cases, the solution will be an exact zero but there might be two roots, and the solution corresponds to the lowest value. Computationally, it will be necessary to identify the discontinuity point, the sign of the discontinuity, and the possibility of multiple zeroes. When the solution coincides with the discontinuity value, the firm's choice of debt b' may raise more resources than necessary to meet the needs of liquidity, and this residual has to be apportioned accordingly, as dividends in the no-default case, and as payments to suppliers in the delinquency case. When the solution is a root, it is bracketed and found using simple bisection. When using extreme-value shocks in the firm's decisions, however, the discontinuities in the price of debt are much mitigated and we have found there is only one root.

Debt pricing function. In this model, the price of debt q^{ND} and q^x in (19) and (20) and, therefore, the resources raised via borrowing may in general be discontinuous in the level of debt chosen b'. This discontinuity occurs at a b' that appears to coincide with the value leading to the discontinuity in the choice of input tomorrow $x(b', ...)$. This discontinuity could in principle be a jump or a drop: a jump as the threshold implies a drop in the probability of bankruptcy on debt since delinquency means operating profits will increase and release cash for repaying bank debt; a drop as the punishment for delinquency may raise the probability of bankruptcy if the productivity punishment for delinquency is sufficiently large. Therefore the price of debt is in general discontinuous with an indeterminate sign. Nonetheless, the presence of EV shocks appears to remove this complication in practice.

C.5 Internal calibration procedure

Since a number of targeted moments will depend directly on the price of inputs p, it is efficient to control p in order to meet those targets, and then choose deep parameters to be consistent with the chosen p.[34]

The steps of the procedure, in outline, are as follows: (i) Set p, and the five deep parameters τ, η, $\tilde{\nu}$, λ, and c_F; (ii) Solve for the firms-lenders equilibrium outcomes Eq. (2) to (8), (19) and (20); (iii) Find entrants' distribution and the value of entry in Eq. (12) and (11); (iv) Solve for the distribution of firms as in Eq. (13) to (16); (v) Calculate trade-credit default rate in Eq. (10); (vi) Calculate target moments (debts, defaults, etc) and check against data; (vii) Update p and parameters τ, η, $\tilde{\nu}$, λ and c_F; Back to point (ii) and repeat until best match to data; (viii) Back out ξ^E via free entry condition Eq. (9); (ix) Back out w via input pricing Eq. (1), the number of firms N by labour market clearing Eq. (28), consumption via clearing in final-goods Eq. (29) and, finally, the parameter B to match the target for employment via the household's optimality condition solving Eq.(23).

D Appendix: Equilibrium functions

We describe the properties of equilibrium objects of this economy. We begin with objects pertaining to the first part of a period, before the productivity shocks occur. Figure 8 displays first the distribution of final-goods firms without a delinquency flag $\nu = 0$ over the initial individual states of previous individual productivity and debt (ϵ_{-1}, b). The mass of firms is spread over the domain of productivity and values and asset positions. The bottom panel displays the corresponding optimal choice of intermediate input x. In terms of sign, the amount of input x is an increasing function of initial productivity, thereby inheriting the properties of the first best. The demand for x is generally increasing in the value of debt due, with more curvature at low levels of productivity, except at very large-debt low-productivity positions where it very likely that the firm will liquidate. (These situations are well outside the range of the stationary distribution.)

Consider now decisions at the end of the period, after the realisation of the new shock ϵ. Figure 9 shows firm's outcomes in the case of no defaulting ND in the space of input and debt (x, b), for a given level of the new productivity ϵ. The top graph displays the borrowing function, with positive values where debt is large enough or small enough, U-shaped, with zero debt in a middle region of x, one which would become wider with larger productivity. The flat area beyond the edges is where debt collapses and corresponds to states where the option of not defaulting ND is unfeasible. The dividends policy is also non-monotonic in x, and positive in the inner region where borrowing is zero. These patterns are valid across all realisations of productivity ϵ. Associated with the decision to be delinquent on

[34] In this way, we avoid having to find p as the solution to the zero-profit condition, an implicit non-linear equation, for each trial of ξ^E.

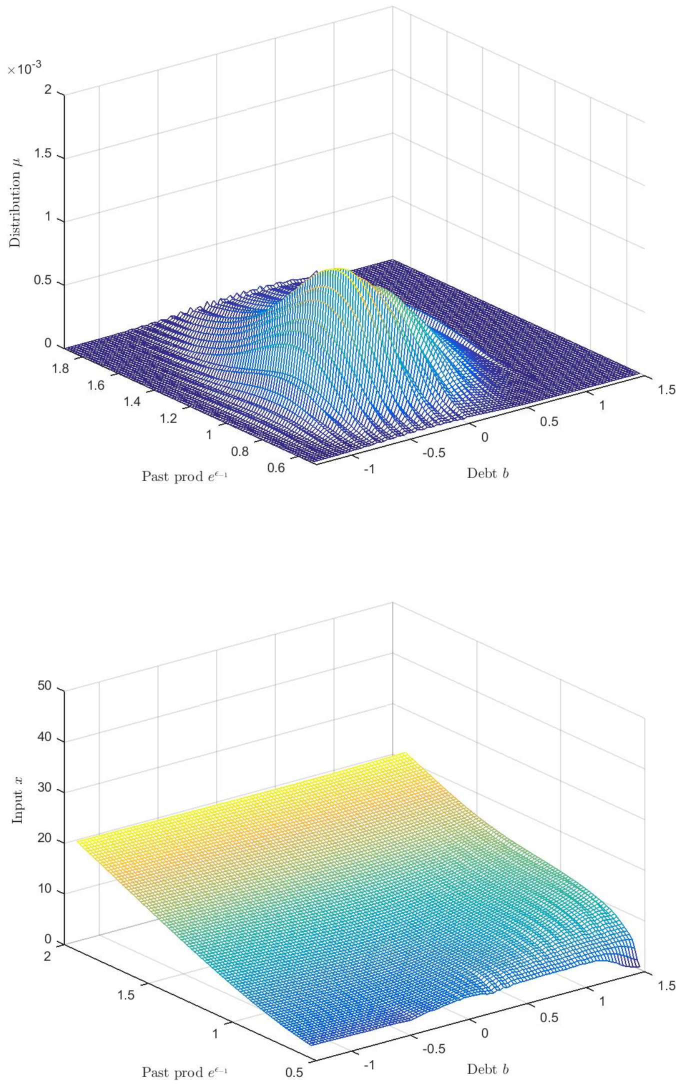

Figure 8: Before shocks: Distribution μ (top) and the demand for inputs x (bottom) as functions of past productivity, $e^{\epsilon_{-1}}$, and debt, b, for firms without a delinquency flag $\nu = 0$. Productivity is normalised to average 1, so debt units are the proportion of output of a final-good firm using the market clearing average level of inputs (equal to 1) and of average productivity.

trade-credit payments, the pattern of borrowing described by the function g^x (not shown) will be qualitatively similar to the one for the no-default case g^{ND} shown in Figure 9, and the shape of the recovery function under delinquency r^x is also similar to the shape of the dividends function π^{ND} already shown.

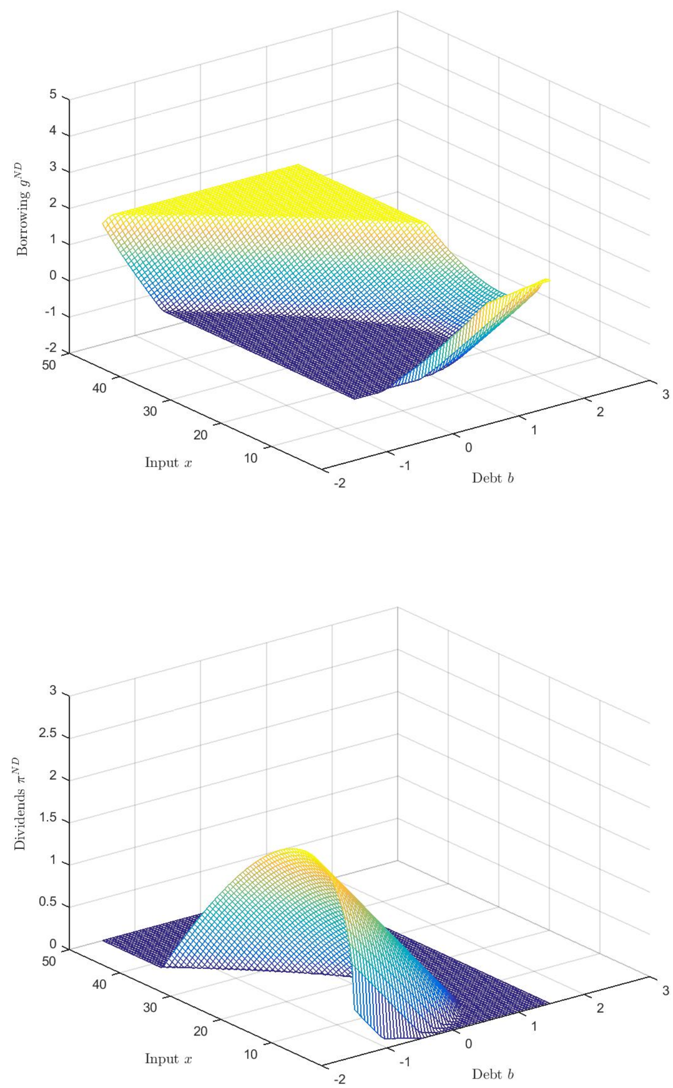

Figure 9: No default case: Policy functions for borrowing, g^{ND}, and dividends, π^{ND}, as functions of debt, b, and inputs purchased, x. Firms with productivity, e^{ϵ}, 12% above the mean, and without a delinquency flag $\nu = 0$.

Figure 10 displays the bankruptcy and delinquency rules in the space of input and debt (x, b), for a low-productivity realisation.The above decision rules on repayment are major determinants of the pricing of debt. Figure 11 displays the price of debt in the space (e^{ϵ}, b') for the cases of no-default, q^{ND}, and delinquency, q^{x}. As expected, debt prices decline in debt borrowed and increase in productivity, and are lower in the event of the firm being currently delinquent.

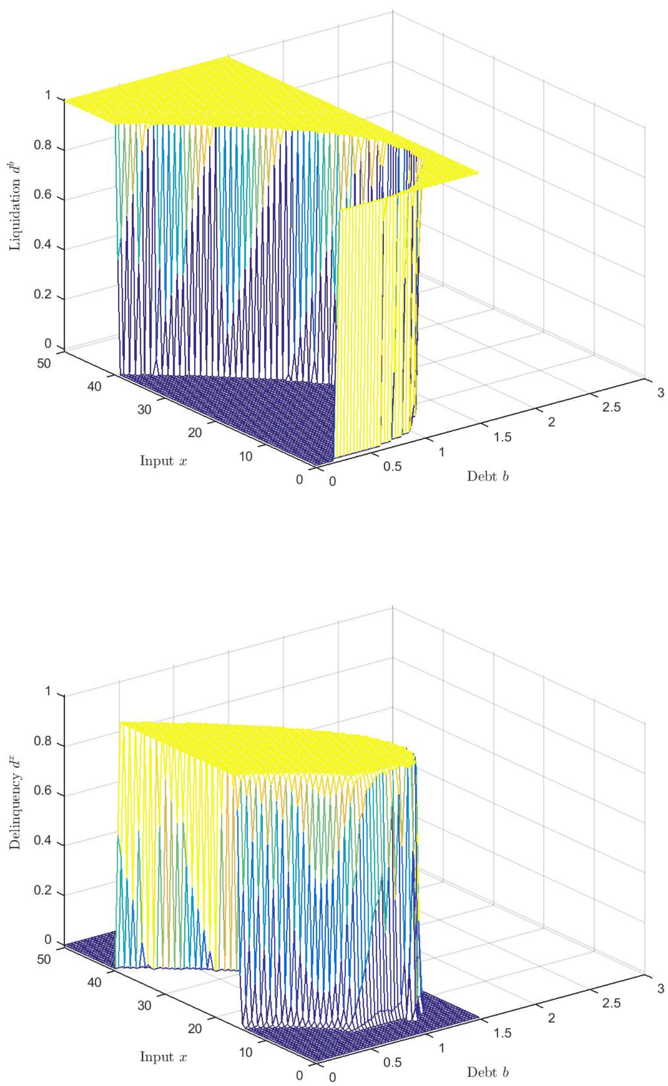

Figure 10: Default rules: Policy functions for bankruptcy, d^b, and delinquency, d^x, as functions of debt, b, and inputs purchased, x. Firms with productivity, e^{ϵ}, 15% below the mean, and without a delinquency flag $\nu = 0$.

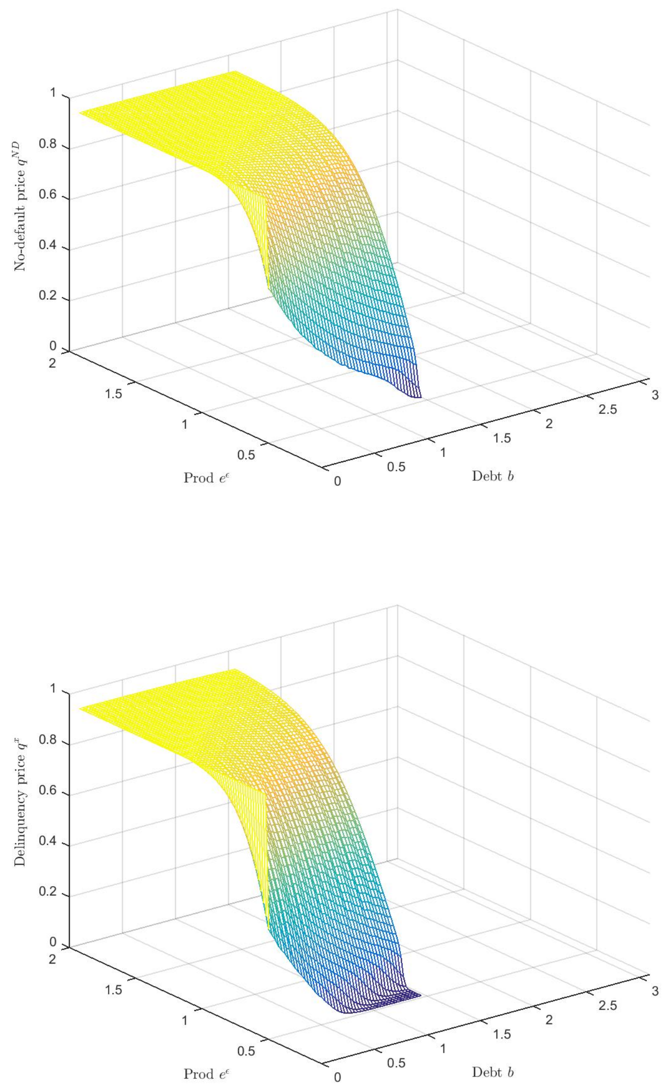

Figure 11: Debt prices: discount prices in no-default case, q^{ND}, and in trade-credit delinquency case, q^{Dx}, as functions of new debt, b', and productivity, e^{ϵ}, for firms without a delinquency flag $\nu = 0$.

E Appendix: Dynamics

E.1 Specification of shocks

Given our parsimonious shock process, we must allow for some latitude in meeting Great-Recession targets, aiming then at a 4% GDP fall on impact and a cumulative fall as close as possible to 9.2%, conditional on the recession length not to exceed three periods.

A financial shock consists of an increase in final-goods firms' fixed cost c_F in the impact period. It is represented by an increase of $c_{F,t}$ in period $t = 0$, observed at the start of period 0, before the realisation of the idiosyncratic shocks, so that forecasts about trade-credit risk are fully updated at this point. After the impact at time 0, we assume that $c_{F,t}$ follows a standard autoregressive process in the difference to its steady state value, that is $c_{F,t} - c_F = \rho_{c_F}(c_{F,t-1} - c_F)$, where $\rho_{c_F} < 1$ is its persistence. We specify a 48% increase, and a persistence of 0.30 (i.e., $c_{F,t=0} = 1.48c_F$ and $\rho_{c_F} = 0.30$) which results in cumulative falls of GDP of 4%, 7% and 9.5% over the three first periods before recovery begins.

The volatility shock consists of an increase in the standard deviation of the innovation to the process of individual firm's productivity, σ_η. It is represented by an increase of $\sigma_{\eta,t}$ in period $t = 0$ that is observed at the start of that period, before the realisation of the idiosyncratic shocks. After the impact at time 0, we assume that $\sigma_{\eta,t}$ follows a standard autoregressive process in the difference to its steady-state value, that is $\sigma_{\eta,t} - \sigma_\eta = \rho_{\sigma_\eta}(\sigma_{\eta,t-1} - \sigma_\eta)$, where $\rho_{\sigma_\eta} < 1$ is its persistence. We specify a 41.5% increase, and a persistence of 0.70: $\sigma_{\eta,t=0} = 1.415$ and $\rho_{\sigma_\eta} = 0.70$ which result in cumulative falls of GDP of 4%, 5% and 8% over the three first periods before recovery begins. Given the series for the standard deviation of the continuous process, to discretise this process, we need to construct a time series for the grid and transition probabilities for ϵ, $\{\mathcal{E}_t, \psi_t\}_{t=0}^T$. We use the idea in Fella, Gallipoli, and Pan (2019) to adapt Tauchen's method to time varying-processes.[35] The resulting productivity distributions obtained with this procedure typically have larger mean relative to the steady state. This should have been expected, given that mean-preserving spread in $\log(x)$ raises mean of x. This is corrected by slightly adjusting ex-post values in grid for productivity to keep the mean unchanged. Notice this tempers the increase in the standard deviation but only to a very small proportion.

The TFP shock consists of a reduction in aggregate productivity z of final-goods firms in the impact period. It takes the form of a reduction of z_t in period $t = 0$, observed at the start of period 0, before the realisation of the idiosyncratic shocks, so that forecasts about trade-credit risk are fully updated at this point. After the shock, we assume that z_t follows a standard log autoregressive process, $\log z_{t+1} = \rho_z \log z_t$, where $\rho_z < 1$ is its persistence, so it returns to its stationary value of $z = 1$ given time. We specify a 1.8% reduction, and 0.80 persistence, that is $z_0 = 0.982$ and $\rho_z = 0.70$, resulting in cumulative falls of GDP of 4% and 6% over the two first periods before recovery begins.

E.2 Solution method

The economy is initially at the baseline stationary equilibrium. Since we are considering the adjustment as a small open economy, the final steady state will differ from the initial

[35] They are interested in a life-cyle setting starting from draws over the distribution of innovations. Since we start from the model's stationary distribution, we adapt their fortran module initialisation, check that it

one in terms of national debt D and corresponding drop in consumption, a wealth effect. Characterising the transition requires solving explicitly for the evolution of, among other variables, the distribution of firms and trade credit default. The procedure to obtain the transition over periods $t = 0, 1, ..., T$ consists of an outer loop in the price of inputs, an inner backward step going from period T to 0 to characterise the firms' and lenders' policy functions and debt price functions, and price of inputs when $m > 0$, and an inner forward step going from period 0 to T to find aggregate quantities and default rates that satisfy the equilibrium conditions, and the price of inputs when $m = 0$. In outline, the main steps are as follows:

1. Guess the new level of consumption c.
2. Guess a path for input price $\{p_t\}_{t=0}^{T}$.
3. Guess a path for aggregate entry rate $\{m_t\}_{t=0}^{T}$.
4. Backward step: Starting at the terminal steady state at $t = T$, proceed backwards to obtain paths for functions $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=1}^{T}$ and $\{q_t\}_{t=1}^{T}$, by solving the equilibrium between final-goods firms and lenders and the free entry condition, and updated input prices in $\{p_t\}_{t=0}^{T}$ when $m_t > 0$ via the free entry condition.
5. Forward step: Given the above paths $\{p_t\}_{t=0}^{T}$ and $\{g_t, x_t, d_t, r_t, \mu_t^E\}_{t=0}^{T}$, and $\{m_t\}_{t=0}^{T}$ proceed forwards to obtain the paths $\{\theta_t, \eta_t, w_t, l_t, \mu_{t+1}, N_{t+1}\}_{t=0}^{T}$, and updated $\{m_t\}_{t=0}^{T}$ and $\{p_t\}_{t=0}^{T}$ that satisfy the corresponding equilibrium conditions, and updating p_t via the input markup condition when $m_t = 0$. Back to 4 until convergence.
6. Obtain the path of national debt D_t, and update consumption from market clearing at the new steady state. Back to 2.

The details of the two loops in steps 4 and 5 above are as follows.

Backward loop:

1. Initialise $q = q_T$ and $W = W_{T+1}$.
2. Let $\rho = \rho_T$ and $Q = Q_T$.
3. Set $t = T$.
4. When $m_t > 0$, find p_t as p that solves free entry (9):
   - Firms Eq. (2) to (8): $p, \rho, q, W \to \tilde{W}$.
   - Free entry Eq. (12), (11), (9): $\tilde{W}, q \to \mu^E, W^E$. Update p and iterate.

   Record time series: $p_t = p$.
   When $m_t \leq 0$, update $p = p_t$. Record time series: $p_t = p$ (although not strictly needed.)
5. Firms' policy functions Eq. (2) to (8): $p, \rho, q, W \to W, x, g, \pi, r, d$.
   Record time series: $W_t = W$, $r_t = r$, $g_t = g$, $\mu_t^E = \mu^E$, $d_t = d$, $x_t = x$.
6. Find debt price functions q_{t-1} as q that meets lenders zero profits at $t - 1$:
   - Let $Q = Q_{t-1}$ and $\rho = \rho_{t-1}$.
   - Lenders Eq. (19) and (20): $Q, d, x, r \to q$.

   Record time series: $q_{t-1} = q$
7. If $t > 0$, update $t = t - 1$ and back to step 4.

Forward loop:

reproduces our stationary distribution after two iterations, and in the third iteration we introduce the shock.

1. Initialise $N = N_0$ and $\mu = \mu_0(:)$.
2. Set $t = 0$.
3. Let $g = g_t$, $r = r_t$, $d = d_t$, $x = x_t$, and $p = p_t$, $\mu^E = \mu_t^E$, $c = c_t$, $m = m_t$, $N = N_t$.
4. When $m_t > 0$, loop over m for labour market clearing:
   - Post-entry distribution: $\mu, \mu^E, N, m \to \hat{\mu}$ by Eq. (16).
   - Delinquency: $\hat{\mu}, d, x, r, d, p \to \theta$ by Eq. (10).
   - Input pricing: $p, \theta \to w$ by Eq. (1).
   - Labour supply: c, w, $\to L$ from (23) via (27).
   - Labour clearing: $\hat{\mu}, N, x, L \to m$ by Eq. (28). Back to top.

   Record time series: θ_t, L_t, m_t, w_t.
   When $m_t \leq 0$, no loop required to find p:
   - Set $m_t = m = 0$.
   - Post-entry distribution: $\mu, \mu^E, N, m = 0 \to \hat{\mu}$ by Eq. (16).
   - Labour clearing: $\hat{\mu}, N, x, m = 0 \to L$ by Eq. (28).
   - Labour supply: c, L, $\to w$ from (23) via (27).
   - Delinquency: $\hat{\mu}, d, x, r, d, p \to \theta$ by Eq. (10).
   - Input pricing: $w, \theta \to p$ by Eq. (1).

   Record p_t, and θ_t, L_t, m_t, w_t.
   Record time series: $c_t = c$.
5. Population dynamics: $N, m, \mu, \mu^E, x, d \to N'$ by Eq. (13).
   Record time series: $N_{t+1} = N'$.
6. Distribution dynamics: $m, \mu, \mu^E, g, x, d \to \mu = \mu N_t / N_{t+1}$ by Eq. (14), (17), (18), and (15).
   Record time series: $\mu_{t+1} = \mu$.
7. If $t < T$, update $t = t + 1$, and back to step 3.

The updating of p within point 4 of the forward loop for the case $m_t \leq 0$ may need some damping for convergence.

Then we shift time indexes so the shock corresponds to the model's impact date 0.