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Welfare implications of endogenous credit limits with bankruptcy

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Abstract

This paper studies the aggregate welfare consequences of changes in the prescribed penalty for personal bankruptcy and in social insurance policies when borrowing limits may respond to these changes. It uses a dynamic general equilibrium model of an exchange economy with incomplete markets and a continuum of agents. The borrowing constraint and the risk of default are endogenous, and the default penalty restricts an individual's access to the markets for a fixed period of time. The effect on the stationary equilibrium of an exogenous reduction of 1 and 2 years in this exclusion period is explored quantitatively. For comparison purposes, the same experiment is carried out under the assumption made in related studies that the borrowing limit is fixed. A small welfare loss follows in either case. In contrast, in a small open economy, welfare may increase substantially but only if the borrowing constraint is endogenous. Similar results follow from an exogenous change in social policy that reduces individual income variability.

JEL classification: D52; D90; G10; G33

Keywords: Borrowing constraints; Incomplete markets; Default; Bankruptcy

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1. Introduction

A large fraction of U.S. families are liquidity constrained, and available estimates point to a substantial effect of credit limits in containing household' levels of debt.[1] Thus borrowing constraints seem to be a pervasive feature of financial markets. Recent research shows their existence may be important for a variety of issues in macroeconomics and finance when financial markets are incomplete.[2] But what determines changes in the level of these borrowing constraints? Are these changes important to understand the consequences of policies or other events on the economy?

The aim of this paper is to take a step towards assessing the significance for the economy of the endogenous determination of borrowing constraints. It pursues two objectives to this end. First, it seeks to set up a theoretical model where the borrowing constraint is determined endogenously. Second, it intends to study within this model the role of the response of the borrowing constraint for the positive and welfare consequences of changes in economic factors and institutions. Attention will be drawn to the institutions or rules that deal with financial default since there is compelling evidence that these factors influence the availability of credit.[3] Social insurance policies – such as unemployment compensation or redistributive taxes – will also be considered since they presumably have consequences for borrowing decisions and the repayment of debts.

The paper studies the equilibrium determination of the borrowing constraint (henceforth also BC) in an economy where financial intermediaries behave optimally and bankrupt individuals are excluded from the markets for a fixed period of time. The analysis is based on a version of Huggett (1993)'s dynamic competitive general equilibrium model of incomplete markets with idiosyncratic risk. In that model, a bond is the only asset agents can trade in order to partially insure consumption subject to an exogenous borrowing limit. There is thus a single type of contract offered to all borrowers irrespective of their individual types or contract size. The present paper departs from the assumption of individual commitment to meet debt repayments. An agent can declare bankruptcy and see her debts discharged. Under these circumstances, the borrowing limit arises endogenously to reflect the factors influencing individual default decisions and the corresponding response of banks.

[1]Hall and Mishkin (1982) and Jappelli (1990) measure the incidence of liquidity constraints. Gross and Souleles (2002), Cox and Jappelli (1993), and Grant (2004) estimate the effect or removing credit limits on levels of debt.

[2]For example, see Zeldes (1989) and Deaton (1991) on consumption and saving; Aiyagari (1994) on wealth distribution; Heaton and Lucas (1996) on asset prices; Aiyagari (1995), and Aiyagari and McGrattan (1998) on fiscal policy; Domeij and Floden (2001) on labor supply.

[3]Gropp et al. (1996) find that state personal bankruptcy exemptions have a significant, positive effect on the probability that households will be turned down for credit or discouraged from borrowing. Grant (2001) finds similar results. Berkowitz and White (2002) show that the supply of credit falls when non-corporate firms are located in states with higher bankruptcy exemptions.

The cost to a borrower for declaring bankruptcy consists of the exclusion from future trades for a certain period of time.[4]

The model is calibrated to grossly match U.S. observations, including interest rates and debt levels. For the parametric settings considered there is no default in an equilibrium with endogenous BC. Banks set such tight borrowing limits because – given the substantial presence of high-risk customers in a debt-constrained state and the thin intermediation spread – the positive risk of default makes it unprofitable extending individual credit lines any further. The effect on the stationary equilibrium of exogenous changes in two parameters of the model are investigated numerically. The first is a reduction in the length of the period of exclusion from financial markets that follows an individual's bankruptcy. When this prescribed punishment is eased, banks tighten up the borrowing limit and ex-ante welfare decreases, the fall in interest rates notwithstanding. The same experiment is repeated but holding constant the borrowing constraint. Although in this case individuals will be more inclined to borrow and default in bad states, the rise in the interest rate and the risk premium also causes a net welfare loss. In either case, a 1 and 2-year shortening of the default penalty lead to comparably modest welfare reductions of between 0.2% and 0.5% in equivalent consumption units. In contrast, for a small open economy the implications can be very different since there is wider room for shifts in the wealth distribution. In this case, with an endogenous BC welfare increases whereas with an exogenous BC welfare declines, and these changes can be quantitatively significant. The gains can be over 1% in equivalent consumption units. The second exogenous change is a mean-preserving reduction in the variability of individual income realizations. In spite of the direct risk-sharing benefits of this change, the consequences are strikingly similar to those of the reduction of the default penalty, both qualitatively and quantitatively.

This paper's contribution is one approach to the endogenous determination of the borrowing constraint with default risk in a familiar class of general equilibrium models, and the analysis of a range of welfare effects of policies under various assumptions. There are papers with endogenous borrowing constraints in incomplete-market economies with individual risk. Zhang (1997) also derives the constraint based on the threat of exclusion from trade if default takes place but, unlike the present paper, the exclusion is permanent and there are only two types of agents. Therefore that model cannot address the questions dealt with here. Cárceles-Poveda and Abraham (2005) extend this analysis to include capital accumulation and state-dependent trading limits.

Chatterjee et al. (2005) study contracts whose price or interest rate depends on the loan size. The interest rate schedule reflects the varying degree of default risk arising from idiosyncratic shocks to individual preferences and productivity. Although there is positive default on some contracts, banks can still balance their books as a higher risk also commands a higher interest rate. The higher interest charged on larger loans is what in effect limits the individual demand for credit, but no individual is credit

[4] U.S. law allows credit bureaus to report past bankruptcies up to 10 years old. Musto (1999) finds that this 'bankruptcy flag' has a big effect on credit access.

constrained. Livshits et al. (2003) and Athreya and Simpson (2005) propose models with similar features. In the present paper, there is instead a single lending rate, irrespective of the contract size or individual type, and a single credit limit. The endogenous credit limit is set by financial intermediaries to reflect the risk of default arising from the realization of individual (productivity) shocks. Unlike the other papers, here some individuals are effectively credit constrained. In fact, the borrowing constraint rules out default in the numerical experiments studied. On the other hand, those other papers deal with a small open economy since they set exogenously the risk-free interest rate. The present paper determines the market-clearing interest rate in equilibrium and can therefore assess important general equilibrium effects. For example, the gains to a shorter punishment period are far smaller in the closed economy than in the open economy. Finally, the other papers cited analyze the consequences of bankruptcy regulations that can be related to the experiments performed in the present paper.

The welfare impact of changes in personal bankruptcy law is also the subject of Athreya (2002) and Li and Sarte (2002). These models share many fundamental characteristics with the one used here, except for their assumption of exogenous borrowing constraints. Therefore they cannot account for the effect implied by the response of borrowing constraints which is the subject of the present paper. These papers run experiments which are similar to the experiment with exogenous borrowing constraint also considered in the present paper for comparison purposes. The present findings would suggest that the welfare results in those works should not change a lot with endogenous BC, but that the positive implications, say for the interest rate, might be quite different. On the other hand, the role of social insurance policies in the presence of default risk have been also considered in Krueger and Perri (2001) in the form of redistributive taxation. Like this paper, they study trading constraints with a continuum of agents. Unlike the present paper, they have a complete rather than incomplete set of insurance contracts, impose rather than derive the participation constraint, and characterize equilibria as constrained efficient allocations rather than directly. Both we and they find that redistribution leads to ex-ante welfare losses. Athreya and Simpson (2005), already mentioned above, deals with the related issue of unemployment insurance in a richer labour market setting and reach similar conclusions.

Section 2 sets out the model and the equilibrium concept, and discusses its novel aspects. Section 3 describes the numerical benchmark and characterizes the properties of the associated equilibrium. Section 4 contains the numerical experiments. Section 5 concludes.

2. The model and equilibrium

This paper studies an exchange economy with incomplete markets where borrowing constraints emerge as the choice of financial intermediaries. The section sets out the model and then defines the equilibrium.

2.1. The model

There is a continuum of individual agents with total mass equal to one. Preferences are defined over stochastic processes for consumption, c_t, and represented by the utility function

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\frac{c_t^{1-\sigma}}{1-\sigma}\quad \text{where } \beta\in(0,1),\ \ \sigma>1,$$

where E is the expectation operator at time 0. Each period each agent receives an endowment y of a perishable good. The set of possible endowments is $Y=\{y_1,y_2\}$ with $y_1>y_2$. The individual endowment follows a Markov process with stationary transition probabilities $\pi(y'\,|\,y)$ for $y',y\in Y$.

An agent is either permitted to borrow and lend or denied access to financial markets. Let z_t denote the number of periods before the agent is given access to markets as of time t. It takes values in the set $Z=\{0,1,2,\ldots,T\}$. When $z_t=0$ the agent can trade a one-period bond or credit balance. Let b_{t+1} denote the individual amount of bonds held between periods t and $t+1$, and q_t the price of one such bond. There is a credit limit or borrowing constraint $\underline{b}$ so that b_{t+1} belongs in $[\underline{b},\overline{b}]$, where $\overline{b}$ is an inconsequential upper bound. An agent with $z_t=0$ may decide either not to repay her negative credit balances at any time by choosing the default variable $d_t=1$, or, otherwise, stay in trade by choosing $d_t=0$. In the former case, the agent will be excluded from trade for T periods as $z_{t+1}=T$ and $z_{t+s}=z_{t+s-1}-1>0$ for $s=1,\ldots,T$. Agents with $z_t>0$ are excluded from trade and cannot hold any credit balances. The exclusion-time penalty, T, is assumed to be given.

Trade in bonds or credit balances takes place through competitive banks or financial intermediaries. A representative bank takes deposits from agents holding positive balances and lends to agents in short positions. It is assumed that banks do not screen individual borrowers' types even though some types may be more likely to default than others. The conditions of credit, including the interest charged on loans, $1/q-1$, and the borrowing limit $\underline{b}$, are thus the same for all individuals. Because of the risk of default on loans, there is a spread between the interest on loans and on deposits denoted by λ. This financial setting resembles today's securitized mortgage markets or securitized credit card markets where buyers of the asset receive a pro-rata share of all different sellers' deliveries.[5] The level of the credit limit $\underline{b}$ is determined by financial intermediaries who seek to maximize profits by balancing the volume of credit extended against the risk of default. Free-entry in financial intermediation means that banks balance their books. Therefore, the price q of

[5]This *anonymity* is characteristic of the research on bankruptcy in general equilibrium in Dubey et al. (2005). It is also used in other papers like Li and Sarte (2002) and Athreya (2002). In Chatterjee et al. (2005) contracts are instead type and size-contingent. There is evidence in Edelberg (2003) that risk-based pricing has been increasingly in use since the 1990s, especially for secured loans such as mortgages and auto loans. For unsecured general consumer loans, credit card loans, and education loans, results are more mixed though. In particular, the loan balance does not appear to be significant for the interest on these contracts (see Edelberg, 2003, Table 12). Thus to a first approach the assumption made here to study the unsecured credit market need not be grossly misleading.

bonds and the spread λ are determined competitively so that the supply and demand for credit are equalized, and banks make zero profits. Individual agents take the spread, the price of bonds, and the borrowing limit as given.

2.2. Equilibrium

This paper will study equilibrium situations where the price of bonds, the loan-deposit spread, and the credit limit are constant over time. The individual state space is $S \equiv B \times Y \times Z$ with elements $s = (b, y, z) \in S$ and $\mathscr{A}_S$ its Borel σ-algebra. The aggregate state then consists of a probability measure Φ over S that describes the distribution of individual types. In a stationary equilibrium this distribution must be constant. A stationary equilibrium can be formulated recursively. Given T and the rest of parameters, an equilibrium is a probability measure Φ on the measurable space $(S, \mathscr{A}_S)$, a price of bonds q, a spread λ, a credit limit $\underline{b}$, a value function $v(.,.,.)$, and decision rules for bonds $b'(.,.,.)$ and defaulting $d(.,.,.)$, such that:

- *Individual choices*: Given q, λ, and $\underline{b}$, the functions $b'(.,.,.)$, $d(.,.,.)$, and $v(.,.,.)$ solve the problem

$$
\begin{aligned}
v(b, y, z) = \max_{b', d} \quad & u(c) + \beta \sum_{y' \in Y} \pi(y' \mid y) v(b', y', z') \\
\text{s.t.} \quad & b' \in [\underline{b}, \overline{b}], \quad d \in \{0, 1\} \\
& c + qb' = y + (1 - d)(b - \max\{0, \lambda b\}) \\
& b' = 0 \quad \text{if } z > 0 \text{ or } z = 0 \text{ and } d = 1 \\
& z' = \begin{cases} z & \text{if } z = 0 \text{ and } d = 0, \\ T & \text{if } z = 0 \text{ and } d = 1, \\ z - 1 & \text{if } z > 0. \end{cases}
\end{aligned}
$$

- *Bank's maximization*: Given λ, Φ, and $d(.,.,.)$, the chosen BC is the smallest value $\underline{b}$ that solves [6]

$$
\max_{\underline{b}} \left\{ - \int_{S: b \geqslant \underline{b}} \min\{0, b\}(\lambda - d(s))\, \mathrm{d}\Phi \right\}.
$$

- *Market clearing*:

$$
\int_S b'(b, y, z)\, \mathrm{d}\Phi = 0.
$$

[6] This is equivalent to maximizing $-\int_{S:b \geqslant \underline{b}} [\min\{0, b\}(1 - d(s)) + \max\{0, b\}(1 - \lambda)]\, \mathrm{d}\Phi$ subject to $-\int_{S:b \geqslant \underline{b}} \min\{0, b\}\, \mathrm{d}\Phi \leqslant \int_{S:b \geqslant \underline{b}} \max\{0, b\}\, \mathrm{d}\Phi$.

- *Free entry*: Banks make zero profits or λ satisfies

$$\int_S \min\{0, b\} d(s)\,\mathrm{d}\Phi = \lambda \int_S \min\{0, b\}\,\mathrm{d}\Phi.$$

- *Stationary distribution*:

$$\Phi(A) = \int_S Q(s, A)\,\mathrm{d}\Phi \quad \text{for } A \in \mathscr{A}_S,$$

with $Q : S \times \mathscr{A}_S \to [0, 1]$ being the transition function derived from the decision rules b' and d, and the transition probabilities $\pi(y' \mid y)$.

The second condition states the problem of the representative bank. With any positive spread λ, it can be seen that the bank will seek to extend the volume traded by loosening the credit limit (i.e. increase $-\underline{b}$) as long as default $d(.,.,.)$ does not increase 'too much'. In particular, as long as default remains zero, the bank will always find it profitable to keep increasing the borrowing limit $-\underline{b}$. The requirement that the constraint be as loose as possible is thus of no consequence in this case. If the spread is instead zero, it is clear that the bank will want to restrict credit in such a way that default remains zero. However, since $\lambda = 0$ the bank's objective will be flat on the region of $\underline{b}$ for which there is no default and profit maximization alone leaves the outcome indeterminate. Hence the extra requirement that the bank sets the loosest debt limit.[7] The characterization of the bank's problem will be discussed further later. The fourth condition states that in a free-entry equilibrium the spread λ must match the default rate, measured by the value of unpaid debts as a proportion of total debt.

In this definition the state space has three variables, including the bankruptcy status z. However, for any $z \neq 0$ the household is in autarky and her decision problem is trivial. Thus an equilibrium can be written in a more manageable form by regarding it as a situation where a certain participation constraint holds for a certain type of agents in S. Let $S_0 \subset S$ denote the set of types that are allowed to trade ($z = 0$) and who do not default ($d(s) = 0$). With this notation, the equilibrium can be characterized as follows. For given T, an equilibrium consists of $\underline{b}$, q, λ, S_0, Φ, $v(.,.,0)$, and $b'(.,.,0)$ such that:

(i) Given $\underline{b}$, S_0, q, and λ, for each type $(b, y, 0) \in S_0$,

$$v(b, y, 0) = \max_{b' \in [\underline{b}, \bar{b}]} \quad u(c) + \beta \left[\sum_{y':s' \in S_0} \pi(y' \mid y) v(b', y', 0) + \sum_{y':s' \notin S_0} \pi(y' \mid y) v^{\mathrm{AU}}(y') \right]$$
$$\text{s.t.} \quad c + qb' = y + b - \max\{0, \lambda b\}.$$

[7]This is not unlike the notion of borrowing constraints 'which are not too tight' studied in Alvarez and Jermann (2000) or Kehoe and Levine (2001), and common in the literature on contract enforcement. This is also assumed in Zhang (1997). Instead of assuming it, one could as well introduce an exogenous fixed cost of intermediation to exactly the same effect. That would cause a profit-maximizing bank to behave as postulated here. In fact, this solution will be equivalent to the present formulation if the fixed cost is chosen to be arbitrarily small so that one can ignore its impact on the interest spread λ and on the market-clearing condition.

with the default continuation value

$$v^{\mathrm{AU}}(y) \equiv E\left[\sum_{t=0}^{T-1} \beta^t u(y_t) + \beta^T v(0, y_T, 0) \mid y_0 = y\right].$$

(ii) *Bank's maximization*: Given λ, Φ, and S_0, the choice of $\underline{b}$ is the smallest value that maximizes

$$-\lambda \int_{S:b \geqslant \underline{b}} \min\{0, b\}\,\mathrm{d}\Phi + \int_{S \backslash S_0 : b \geqslant \underline{b}} b\,\mathrm{d}\Phi.$$

(iii) *Market clearing*:

$$\int_{S_0} b'(b, y, z)\,\mathrm{d}\Phi = 0.$$

(iv) *Free entry*:

$$\int_{S \backslash S_0} b\,\mathrm{d}\Phi = \lambda \int_S \min\{0, b\}\,\mathrm{d}\Phi.$$

(v) *Stationary distribution*:

$$\Phi(A) = \int_S Q(s, A)\,\mathrm{d}\Phi \quad \text{for } A \in \mathscr{A}_S.$$

(vi) *Participation constraint*: For and only for $s \in S_0$

$$v(b, y, 0) \geqslant v^{\mathrm{AU}}(y).$$

The advantage of this definition over the original one is that the consumer's problem becomes simpler as it has to be solved only for agents that do not currently default. The defaulting choice has been replaced by the determination of such a set of agents S_0 through the participation constraint. Note that the value of exclusion is well defined since $v(0, y, 0) \geqslant v^{\mathrm{AU}}(y)$ must hold.

2.3. Characterization

The notable features of this equilibrium are the determination of the non-default set S_0, point (vi), and the borrowing constraint $\underline{b}$, points (ii) and (iv). Consider first the determination of the no-default set S_0. It is useful to express the value and policy functions conditional on the underlying $\underline{b}$ and S_0. Following this convention, the participation value $v(b, y, 0 \mid \underline{b}, S_0) - v^{\mathrm{AU}}(y \mid \underline{b}, S_0)$ is increasing in b. For an agent with income $y \in \{y_1, y_2\}$, one can define $b(y)$ as the value of $b \in [\underline{b}, \overline{b}]$ where this expression becomes non-negative. Clearly, $b(y) \leqslant 0$ since $v(0, y, 0 \mid \underline{b}, S_0) - v^{\mathrm{AU}}(y \mid \underline{b}, S_0) \geqslant 0$. Given the pair $(b(y_1), b(y_2))$, the no-default set S_0 is given by points to the right of this $b(y)$ for each y:

$$S_0 = S_0(b(y_1), b(y_2)) \equiv \{s \in S : z = 0, b \geqslant b(y), \forall y \in Y\}.$$

Default thus occurs among agents with high debts. Clearly S_0 may influence the $b(y)$'s which in turn may affect S_0. To see this, it will prove useful to define the participation value for an agent with income y_i and assets $b(y_i)$ for $i \in \{1, 2\}$ as

$$PART_i(b(y_i)) \equiv v(b(y_i), y_i, 0 \mid \underline{b}, S_0(b(y_1), b(y_2))) - v^{\mathrm{AU}}(y_i \mid \underline{b}, S_0(b(y_1), b(y_2))).$$

With this notation, it is straightforward to establish that in equilibrium $b(y_i)$ must be such that $PART_i(b(y_i))$ becomes non-negative, for $i = 1$ and 2, respectively. Thus in order to find the equilibrium default set S_0 one needs to understand how the $b(y)$'s influence this participation value. For a fixed y_i, there are two effects of changes in $b(y_i)$ on the participation condition $PART_i(b(y_i))$. First, there is the direct impact on the first argument of the value function of participating $v(b(y_i), ., . \mid .)$. The sign of this effect must be positive. Second, there is an indirect effect through the changes in the no-default set, $S_0(., b(y_i))$, which is an argument of both the value of participation and the value of defaulting $v^{\mathrm{AU}}(. \mid .)$. The sign of this effect may well depend on the specific circumstances. On the other hand, changes in the default threshold for other income levels $b(y_j)$ for $j \neq i$, will also affect the participation condition for income y_i via S_0 in ways which are hard to establish at this level of generality. Although the determination of the pair $(b(y_1), b(y_2))$ will be addressed below within specific numerical settings, Fig. 1 is now offered in anticipation of the results. Given the equilibrium $\underline{b}$ and r, Fig. 1 depicts one plausible pattern of the participation value $PART_i(.)$ for the two income levels indexed $i = 1, 2$. The slopes and relative positions represented will be characteristic of

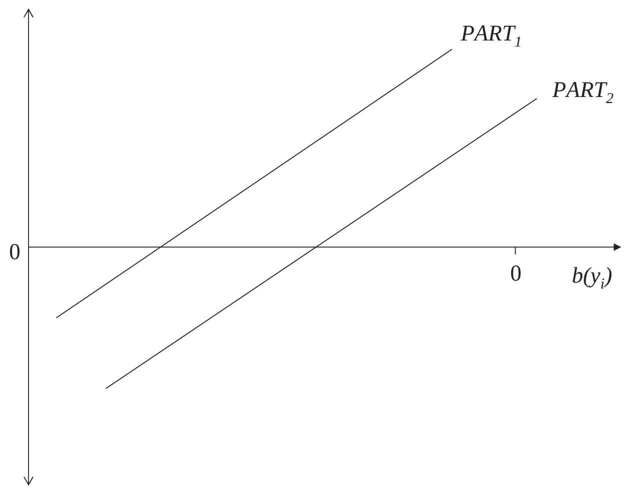

Fig. 1. Determination of default thresholds and the BC.

the numerical settings to be analyzed later. For each income level, the default threshold value of assets $b(y_i)$ is determined by the intersection of the corresponding curve $PART_i$ with the zero axis.

Second, turning to the determination of the borrowing constraint, the bank's problem can now be represented as follows: Given $b(y)$ and Φ, maximize

$$-\int_{S:\underline{b}\leqslant b<b(y)} \min\{0,b\}(\lambda-1)\,\mathrm{d}\Phi - \int_{S:b\geqslant b(y),b\geqslant\underline{b}} \min\{0,b\}\lambda\,\mathrm{d}\Phi,$$

where $b(y)$ stands for the defaulting threshold level of debt defined in the previous paragraph. The first term is negative as it accounts for failing loans. The second term is positive and picks up the gains from good loans. A tighter borrowing limit (i.e., larger $\underline{b}$) does reduce the risk of bad loans but also constraints the volume of profitable good loans. The optimal choice balances these two contrary effects. To characterize this trade-off in the specific model, assume at no loss and like in Fig. 1 that a low-income individual will default at a lower level of debt than the high-income individual, that is $b(y_2)\geqslant b(y_1)$. To begin with, start with a $\underline{b}$ loose enough in the sense that $\underline{b}<b(y)$ for all income levels $y\in\{y_1,y_2\}$. If $\Phi(b,y,0)$ has a positive mass for $b\in[\underline{b},b(y_1))$ the bank will find it profitable to restrict credit (raise $\underline{b}$) at least until default by high-income individuals is ruled out, or $\underline{b}=b(y_1)$. More formally, for $\underline{b}<b(y_1)\leqslant b(y_2)$, the marginal gain to raising $\underline{b}$ is

$$-(-\underline{b})(\lambda-1)[\Phi(\underline{b},y_2,0)+\Phi(\underline{b},y_1,0)]\geqslant 0.$$

Consider now that the credit limit is too tight in the sense that $\underline{b}\geqslant b(y)$ for all income levels. In such a situation there is no default and the bank will not find it profitable to restrict credit any further. More formally, if $\underline{b}\geqslant b(y_2)\geqslant b(y_1)$, the marginal gain to raising $\underline{b}$ is

$$-(-\underline{b})\lambda[\Phi(\underline{b},y_2,0)+\Phi(\underline{b},y_1,0)]\leqslant 0.$$

As long as $\lambda>0$, this expression is strictly negative so $\underline{b}\leqslant b(y_2)$. For the case that $\lambda=0$ this expression holds as a strict equality and, as explained in Section 2.2, that $\underline{b}\leqslant b(y_2)$ will be assumed. This discussion therefore establishes that the optimal borrowing constraint must fall somewhere between these two situations, or $\underline{b}\in[b(y_1),b(y_2)]$. Within this range, the marginal gain to tightening the credit limit reads

$$-(-\underline{b})[(\lambda-1)\Phi(\underline{b},y_2,0)+\lambda\Phi(\underline{b},y_1,0)].$$

It can be seen that if the mass of high-income low-risk individuals in the margin, $\Phi(\underline{b},y_1,0)$, is sufficiently large relative to that of low-income high-risk individuals, $\Phi(\underline{b},y_2,0)$, then the gain to raising $\underline{b}$ will be negative and the bank will be willing to relax the borrowing constraint and extend credit even at the expense of causing some positive default risk, or $\underline{b}<b(y_2)$. The bank benefits from low-risk loans more than it loses on failing loans. This type of cross subsidization requires that $\Phi(\underline{b},y_1,0)>(1/\lambda-1)\Phi(\underline{b},y_2,0)$. Since a realistic risk-spread λ value cannot exceed 0.10, this condition requires in practice that the mass of low-risk individuals be at least severalfold larger than that of high-risk individuals.[8] Thus this condition can be expected to fail if, near the borrowing limit, the relative mass of high-income individuals cannot be made overwhelmingly large. In the specific numerical settings studied later, this will in effect be the case so that the gain to raising $\underline{b}$ will be positive and default will be ruled out, or $\underline{b} = b(y_2)$.[9] In Fig. 1, the borrowing constraint $\underline{b}$ will be determined by the intersection of $PART_2$ with the zero axis, and then $b(y_1) = b(y_2) = \underline{b}$.[10]

The iterative procedure for computing an equilibrium is divided in two main steps: (1) Guess values for λ and $\underline{b}$; (2) Compute the equilibrium except for conditions (ii) and (iv), i.e. compute S_0, q, $v(.,.,.)$, $b'(.,.,0)$, and Φ; (3) Verify the equilibrium is consistent with conditions (ii) and (iv), or update λ and $\underline{b}$, and start back in step 2. Step 2 is similar to solving Huggett (1993)'s model except for the fact that some types of agents may default so the set S_0 must be determined (in Huggett (1993) $S_0 = S$). This requires an extra round of iterations. The specific steps are as follows: (a) Fix a q; (b) Initialize $S_0 = S$; (c) Solve $v(.,.,0)$ and $b'(.,.,0)$; (d) Check that $v(b, y, 0| \underline{b}) - v^{AU}(y| \underline{b}) \geqslant 0$ iff $(b, y, 0) \in S_0$. Update S_0 and go to step c; (e) Compute Φ; (f) Check market clearing. Update q and go back to step b. The equilibria in this paper are situations where the procedure converges to a stationary distribution Φ. A general result on the existence and uniqueness of a stationary equilibrium distribution is not provided in this paper. One will thus rely largely on computational results. In the numerical explorations carried out in this research, convergence to the stationary distribution is not generally a problematic issue, even when a positive level of default is involved (i.e., $S_0 \neq S$). The appendix contains details on the implementation of the computations.

3. The benchmark model

The equilibrium implications of this model will be analyzed numerically. Benchmark values for the parameters have to be selected. This section presents this choice and describes the properties of the corresponding equilibrium.

The parameters of the model related to the process of individual income are (y_1, y_2), $\pi(y_1 \mid y_1)$, and $\pi(y_2 \mid y_2)$. They will take on values based on Kydland (1984) and used in Huggett (1993) to parameterize an economy where one period corresponds to two months and individual risk is associated with changes in unemployment status in the U.S. The upper bound on assets $\overline{b}$ is set so that it is rarely binding. The three remaining parameters are the intertemporal elasticity of substitution σ, the discount rate β, and punishment period for defaulting T. In the

[8]Note that, in the event of default, an individual defaults on all her debts and not only on the marginal line of credit.

[9]The values $\Phi(\underline{b}, y_1, 0)$ and $\Phi(\underline{b}, y_2, 0)$ will be found to be very similar.

[10]Note that the very position of these curves will also change as $\underline{b}$ changes. Also, the curves $PART_i$ are not well defined to the left of $\underline{b}$.

Table 1
Benchmark model

Parameters							
$\pi(y_1 \mid y_1)$	$\pi(y_2 \mid y_2)$	y_1	y_2	β	σ	T	$\overline{b}$
0.925	0.50	1.00	0.10	0.991	1.5	32	4.00
Equilibrium							
r	λ	$\underline{b}$	$b(y_1)$	$b(y_2)$	j_1	j_2	
0.00635	0.00	−5.20	−5.20	−5.20	1	1	

baseline calibration, they will be chosen to match targets for the borrowing limit, the interest rate, and the actual punishment for default.

Regarding the interest rate, the bimonthly borrowing rate $1/q - 1$ will be denoted by r. In annual terms, it should lie between the 5% S&P Index average return and a 15% on credit cards, say 10%. However, since in the model default will be zero, one should adjust this figure for the default rate in the data. An annual default rate around 5–6% is close to the average behavior for corporate bonds and personal unsecured loans, largely credit card lines.[11] The baseline calibration matches an annual interest rate near 4%. There are no precise measures of the period of exclusion after default, T. The U.S. code prescribes that the bankruptcy records be held for 10 years. Athreya (2002), based on anecdotal evidence, considers 4 years of exclusion. Any number within this range would be acceptable.[12] In the benchmark calibration, T is chosen to lie in the region of 5 years. As reported in Chatterje et al. (2005) and Athreya (2002) the maximum level of debt is probably no larger than 1-year's average income, which in the model is 5.3. The choices made in the benchmark calibration imply a debt limit of 5.2. However, the fact that the model delivers zero default, suggests that a tighter constraint could also be reasonable. The baseline parameters are displayed on the top half of Table 1.

The key endogenous variables are the borrowing constraint $\underline{b}$, the risk spread λ, the interest rate r, the default levels of assets $b(y_1)$ and $b(y_2)$ (which characterize S_0), along with the wealth distribution Φ, and the individual asset policy function b'. Attention will be drawn first to the determination of the default decisions, the spread, and the borrowing constraint displayed in the bottom part of Table 1.[13] The

[11]Moody's KMV historical default report documents that over the post-1970 period, default rates and average default losses have reached averages of 6.47% with lower rating categories and a 3.33% for the single B category (see http://riskcalc.moodysrms.com/us/research/defrate.asp). The Federal Reserve Board releases charge-off and delinquency annual rates on unsecured credit card loans around 5% since 1996 (see http://www.federalreserve.gov/releases).

[12]Unlike Athreya (2002), the present model rules out savings during the exclusion period as well as pecuniary costs of default. Moreover, since default will be zero, a borrowing constraint tighter than in the real world could also be reasonable.

[13]The integers j_1 and j_2 displayed here and in other tables below are the index points in the grid for assets used in the computation associated with $b(y_1)$ and $b(y_2)$, respectively.

Table 2
The borrowing limit $\underline{b}$

$\underline{b}$	$\lambda \times 100$	$r \times 100$	$b(y_1)$	$b(y_2)$	j_1	j_2
−5.20	0.000	0.635	−5.20	−5.200	1	1
−5.30	0.100	0.706	−5.30	−5.292	1	5
−5.40	0.110	0.724	−5.40	−5.388	1	6
−5.60	0.115	0.752	−5.60	−5.582	1	7
−5.80	0.124	0.778	−5.80	−5.775	1	8
−6.00	0.130	0.803	−6.00	−5.966	1	9
−6.40	0.140	0.843	−6.40	−6.355	1	10
−6.80	0.150	0.878	−6.80	−6.741	1	11
			....			
−7.60	0.350	1.030	−7.424	−7.506	17	13

interest rate, also displayed in Table 1, and the distribution will be discussed next. Finally, other reasonable parameterizations will be presented.

3.1. The determination of default and the borrowing constraint

In this equilibrium the credit limit just rules out positive default (i.e., $b(y_1) = b(y_2) = \underline{b}$) and the risk spread is zero (i.e., $\lambda = 0$). To see why this is the only equilibrium, Table 2 shows, for various given debt limits $\underline{b}$, the implied values for the interest rate r, the default rules $b(y)$, and the default-risk spread λ. The first row corresponds to the baseline equilibrium. The remaining rows correspond to situations that satisfy all the equilibrium conditions except possibly bank maximization. A consistent feature throughout is that, at the borrowing limit, the (small) mass of individuals in the two income groups is virtually identical (i.e, $\Phi(\underline{b}, y_1, 0) \approx \Phi(\underline{b}, y_2, 0)$). According to the discussion in Section 2.3, it follows that the bank finds it optimal to restrict credit whenever there is positive default (i.e., when $\lambda > 0$). Table 2 shows that any looser-than-equilibrium constraint would lead to positive default by some highly indebted low-income individuals (i.e., $b(y_2) > \underline{b}$) and thus will be inconsistent with an equilibrium. The debt constraint will be raised until it reaches its baseline zero-default value.

Inspection of Table 2 also reveals other features. The default rate increases monotonically as $\underline{b}$ is reduced, and eventually may involve high-income agents. The bottom row, for example, demonstrates that for a low enough $\underline{b}$, the pattern of bankruptcy is reversed in that the high-income individuals become those who start defaulting at a lower level of debt (i.e., $b(y_2) > b(y_1)$).[14]

[14]If saving is permitted after bankruptcy this might be more so. However, it is hard to say whether only that could make the binding individual to be of high income in an equilibrium.

Table 3
The interest rate r

$r \times 100$	DEM	$b(y_1)$	$b(y_2)$	j_1	j_2	$DEF \times 100$
0.560	−0.468	−5.20	−5.200	1	1	0.000
0.600	−0.227	−5.20	−5.200	1	1	0.000
0.635	−0.006	−5.20	−5.200	1	1	0.000
0.670	+0.247	−5.20	−5.198	1	3	0.076
0.750	+0.792	−5.20	−5.192	1	5	0.103
0.850	+1.372	−5.20	−5.183	1	7	0.125
1.000	+1.978	−5.20	−5.176	1	8	0.147
1.500	+2.831	−5.20	−5.138	1	12	0.201

3.2. The interest rate and wealth distribution

Given the equilibrium spread and borrowing limit, the determination of the equilibrium interest rate clears the market for credit. This is standard except for two special features of this model. The first is that the interest rate has an effect of default behavior and could potentially alter the standard monotonicity and/or continuity properties of the excess demand function. The second is that when there is positive default the underlying distribution over levels of assets and income $\Phi(\ldots)$ must account for the flows of individuals into and out of the bankruptcy status. A result on the convergence of the distribution – of the type established in Huggett (1993) – does not exist in this case and this could prevent the characterization of the equilibrium. These two issues are not a concern in practice. To see this, Table 3 displays the excess demand function for bonds DEM, the default rate DEF, and the default rules $b(y_1)$ and $b(y_2)$ generated by various interest rates around the equilibrium. Since r, λ and $\underline{b}$ are held as given, these values may fail to satisfy the equilibrium conditions of market clearing, bank's zero-profit, and bank's optimization.

Observe that default increases with the interest rate which goes counter the rise in the net demand for bonds. The figures in Table 3 demonstrate that the monotonicity and continuity in the excess demand function are preserved even where there are shifts in the default rules. For each given interest rate, the distribution converges without problems to the stationary configuration used to calculate the demand for credit.

3.3. Alternative settings

As discussed before, there are arguably alternative choices of calibration targets which could be just as defensible as those leading to the baseline parameters. To explore the sensitivity of the results, as well as to gain a grasp of the model's implications, some such alternative settings have also been studied. These settings are characterized by the same parameters as the baseline economy, except for σ and T.

Table 4
Other settings

Case	σ	T	$\underline{b}$	$r \times 100$
(1)	1.50	32	−5.20	0.635
(2)		28	−4.70	0.557
(3)		24	−4.30	0.477
(4)	1.10	52	−5.10	0.723
(5)		48	−4.80	0.693
(6)		42	−4.30	0.630
(7)	2.00	21	−5.20	0.478
(8)		17	−4.70	0.436
(9)		15	−4.40	0.275

More precisely, for a given σ, T is calibrated to match a borrowing limit which is approximately 95% (as in the baseline setup), 80% and 70% of the 1-year income mark 5.3, respectively. This is repeated for three alternative values of σ, including the baseline value. Table 4 displays these choices and their implications for the level of the borrowing constraint and the interest rate. In equilibrium, default is invariably zero across all the examples so $\lambda = 0$ and $b(y_1) = b(y_2) = \underline{b}$.

Matching a tighter constraint (i.e., higher $\underline{b}$) requires a softer bankruptcy penalty (i.e. lower T). This induces a greater inclination to default on the part of individuals and prompts a response by the bank in the form of the targeted tighter BC. It follows that the equilibrium interest rate declines.[15] For the baseline $\sigma = 1.5$, the interest rate falls slightly short of the range of targeted interest rates, and the penalty period declines towards the lower region of 4 years used in Athreya (2002). For the lower $\sigma = 1.10$, the interest rate is larger and falls comfortably within the acceptable range. The required penalty T is larger and takes on values in the acceptable upper range, equivalent to between 7 and 10 years. For the higher $\sigma = 2.00$, the interest rate lies in a region corresponding to between 3 and 2 annual per cent, well below the targeted range. The bankruptcy exclusion is also too small, equivalent to between 3 and 2.5 years.

All these settings share the same properties as the baseline case in row (1) discussed in Sections 3.1 and 3.2. An exogenously looser constraint would lead to positive default and a response of the interest rate. The interest rate in turn has an impact on the default rate, yet the net demand for credit is well defined and the distribution always converges to its stationary limit.

[15] This is despite the fact that, for a given borrowing constraint, a softer default penalty creates a greater tendency to borrow and use the bankruptcy option.

4. Numerical experiments

This section studies changes in the exogenous length of the exclusion period that penalizes default, T, and in the relative value of individual income realizations, y_1 and y_2. Each type of change is analyzed under two alternative scenarios. The first corresponds to the equilibrium presented so far in which the borrowing constraint is determined endogenously. In the second scenario, the borrowing constraint is held constant and the default rate adjusts accordingly.[16] The response of various variables under the two scenarios will then be compared. The variables of interest include the interest rate r, the risk premium λ, and a measure of social welfare. Welfare W will be calculated as the expected value function, $v(s)$, over assets b, income y, and credit status z according to the following:

$$W = \int_S v(b, y, z)\, \mathrm{d}\Phi.$$

This is a measure of ex-ante welfare. The proportional change in W in equivalent consumption units relative to the corresponding benchmark will be calculated as $\Delta WC \equiv (W/W^{\mathrm{B}})^{1/(1-\sigma)} - 1$, where W^{B} is the level of welfare in the benchmark equilibrium.

4.1. Bankruptcy penalty

A reduction in T is considered. It can be related to a reform of the bankruptcy code or a change in the practices followed by lenders in their use of individual credit histories.

4.1.1. Endogenous borrowing constraint

The first type of experiment traces the response of the endogenous variables to such a change within the equilibrium concept considered so far. The endogenous variables then include the borrowing constraint $\underline{b}$, the interest rate r, the default thresholds of debt $b(y)$, and the default-risk spread λ. Section A of Table 5 displays the consequences of reducing T by an amount equivalent to 1 and 2 years starting from the baseline equilibrium shown in the first row. As T is reduced, the borrowing constraint becomes tighter and the interest rate declines. The squeeze on the demand for credit explains the response of the interest rate. The default rate remains zero throughout for exactly the same reasons as in the baseline economy, hence the constant values of λ, and the $b(y_i)$'s being equal to the limit.[17] Turning to welfare W, the change in T appears to have negative consequences, albeit quantitatively small. The percentage changes of welfare in equivalent units of consumption, ΔWC, are about 0.2 and 0.5.

[16]More specifically, all the equilibrium conditions are satisfied except (ii), the bank's optimal determination of $\underline{b}$. This type of situation has already been analyzed in the computations leading to Table 2.

[17]For high-income types, $b(y_1)$ is always at the limit regardless and is not displayed.

Table 5
Lower T with endogenous BC, baseline model

T	$\underline{b}$	$\lambda \times 100$	$r \times 100$	$b(y_2)$	j_2	W	$\Delta WC(\%)$
A. General equilibrium							
32	−5.20	0.00	0.635	−5.200	1	−237.569	
26	−4.50	0.00	0.520	−4.500	1	−237.792	−0.19
20	−3.70	0.00	0.310	−3.700	1	−238.164	−0.50
B. Small open economy							
32	−5.20	0.00	0.635	−5.200	1	−237.569	
26	−4.10	0.00	0.635	−4.100	1	−236.789	+0.66
20	−3.00	0.00	0.635	−3.000	1	−236.208	+1.16

In order to interpret the response of aggregate welfare, the underlying changes in utility levels and the wealth distribution must be examined in some detail. Results will be reported for the changes between the baseline equilibrium and the equilibrium corresponding to the lower $T = 26$. Fig. 2 displays the value function $v(b, y, 0)$ for high-income and low-income individuals over levels of assets.[18] The utility at most individual states declines slightly with a lower T. For low-income highly indebted agents, the decline in utility is much more pronounced, as one would expect under more restrictive credit conditions. Fig. 3 depicts the cumulative distribution of high-income and low-income individuals across asset levels. There is a shift of the distribution away from the tails towards the median, whose effect on overall welfare is fairly neutral. Thus the direct changes in utility will be the key to explaining the welfare implications of T.

The shifts in the value functions are caused by the response of the only two variables that bear on individual households: the interest rate and the borrowing limit. Fig. 4 intends to disentangle their respective roles by drawing the value function when only either of these variables changes, given the rest of variables are held at their baseline values. The fall in the interest rate increases utility at every individual state, most visibly at high levels of debt. The tighter credit limit, on the other hand, reduces the utility at every individual state, mainly at high levels of debt, and specially for low-income individuals. This negative effect reflects the more limited possibilities for risk-sharing. Therefore, the reduction of welfare caused by a lower T is an expression of the adverse risk-sharing consequences of more restrictive credit conditions. In general equilibrium, this effect is mitigated, but not overturned, by the lower interest rate.

[18]Hereafter, the figures will represent values for the points on the grids used in the calculation. As described in the appendix, there are therefore more points at the lower range of values for b and one can get the ‘wrong’ visual impression concerning the curvature of the schedules represented. Another observation, pertinent to situations with endogenous BC, is that when different curves are created on different grids only a few points that match values across the grids are represented hence the apparent lack of smoothness in some cases.

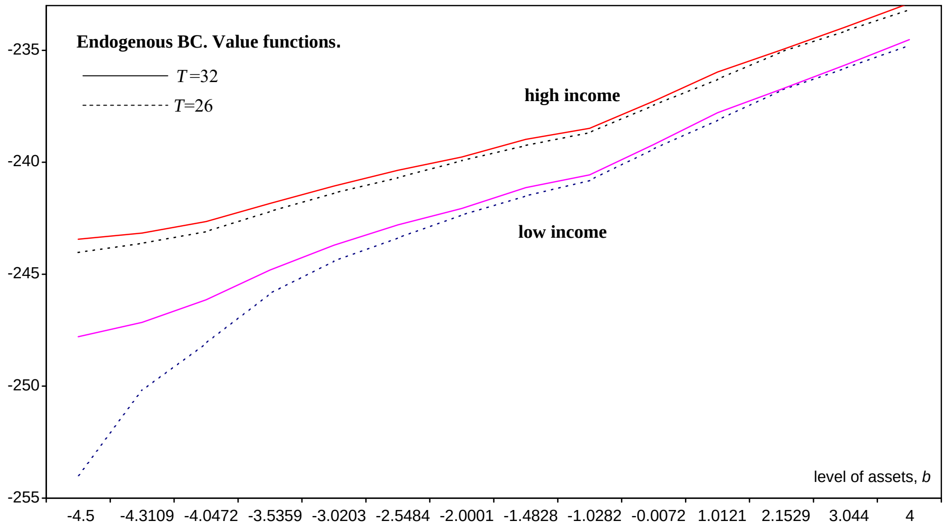

Fig. 2. Endogenous BC. The effect of lower T on the value function.

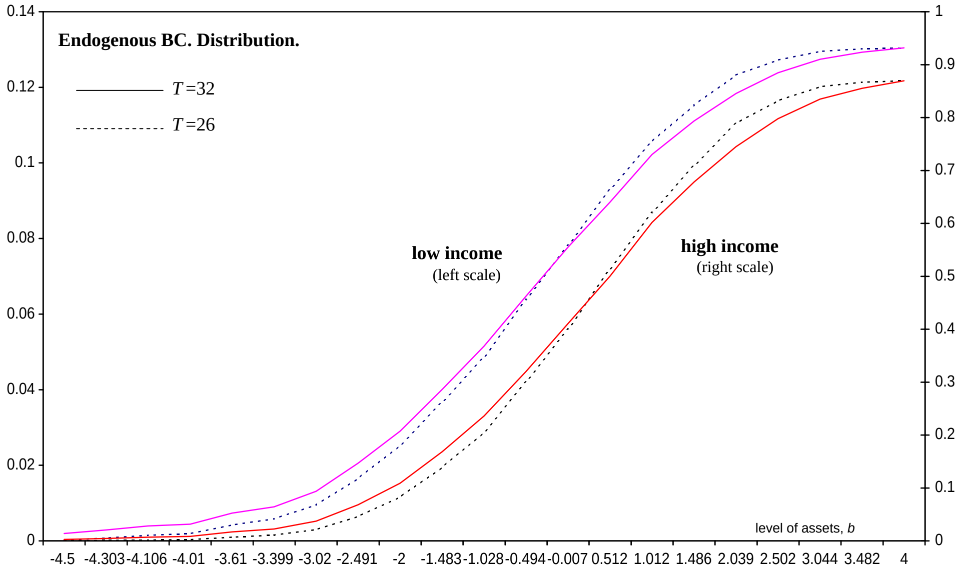

Fig. 3. Endogenous BC. The effect of lower T on the wealth distribution.

In this analysis, the market clearing condition requires that total borrowing and lending match up. As shown in Fig. 3, this probably constraints the scope for shifts in the distribution of individuals across levels of wealth and their implications for welfare. To assess the importance that these implications may have, the same

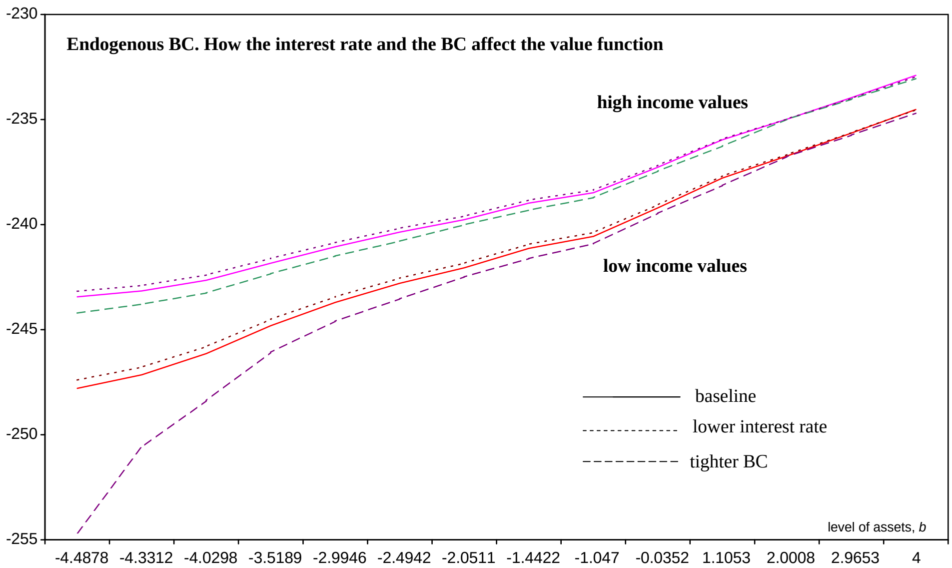

Fig. 4. Endogenous BC. The effect of r and $\underline{b}$ on the value function. Lower interest rate: $r = 0.0052$; tighter borrowing constraint: $\underline{b} = -4.5$.

changes in T will be studied when market clearing need not be satisfied and the interest rate remains constant. These experiments will thus characterize the response of a small open economy. Section B of Table 5 shows the results. The reduction in T leads to an even tighter constraint than when r was let to decline endogenously. The reason is not hard to grasp since, as one learned before, the higher fixed interest rate is associated with a tendency to default more for a given borrowing constraint. The measure of welfare W now shows an increase which, as indicated by ΔWC, is in the order of 0.7% and 1.2% in equivalent consumption units. This contrasts, both qualitatively and quantitatively, with the response of the closed economy. The explanation has to be found in the adjustment of the value function and the wealth distribution to the rise in $\underline{b}$. Figs. 5 and 6 display these responses for the case when T is reduced from 32 to 26. The value function declines, like before, because of the narrower opportunities to smooth consumption through borrowing. The distribution, on the other hand, experiences a notable shift to the right, thus increasing the mass of individuals in high-utility states. This effect on the distribution dominates over the effect on the value function and dictates the overall welfare improvement.[19]

Summing up, when the BC is endogenous the welfare consequences of easing the default penalty T are driven by the implications of the subsequent tightening of the

[19] In the closed economy this distribution effect and its impact on welfare were offset by the opposite force brought about by the lower interest rate, giving rise to the approximately 'welfare-neutral' pattern of change shown in Fig. 3.

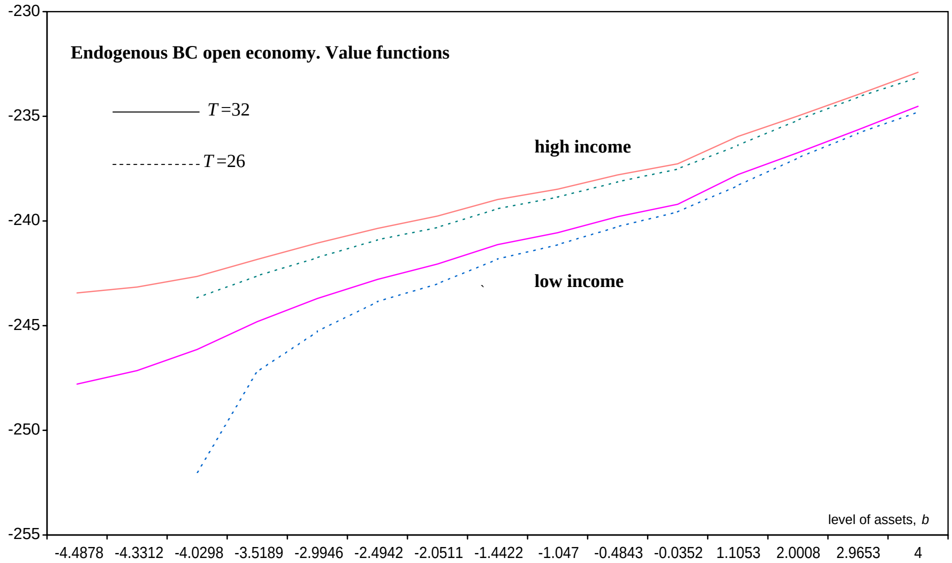

Fig. 5. Endogenous BC in the open economy. The effect of T on the value function.

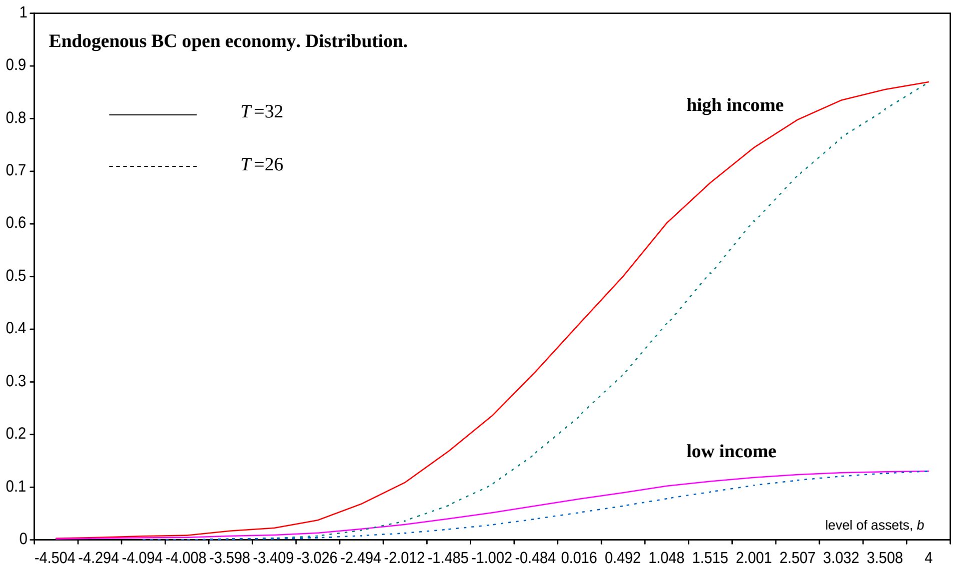

Fig. 6. Endogenous BC in the open economy. The effect of T on the wealth distribution.

borrowing constraint. In general equilibrium, welfare declines because of its adverse risk-sharing implications, but quantitatively the net impact is fairly small. In an open economy, welfare increases because of its beneficial implications for the distribution

Table 6

		General equilibrium				Open economy			
σ	T	$\underline{b}$	$r\times 100$	W	$\Delta WC(\%)$	$\underline{b}$	$r\times 100$	W	$\Delta WC(\%)$
(a) *Lower T with endogenous BC, robustness*									
1.10	52	−5.1	0.723	−1125.880		−5.1	0.723	−1125.880	
	46	−4.6	0.673	−1125.974	−0.08	−4.6	0.723	−1125.485	0.35
	40	−4.1	0.600	−1126.093	−0.19	−4.0	0.723	−1125.049	0.74
2.00	21	−5.2	0.478	−127.461		−5.2	0.478	−127.461	
	15	−4.4	0.275	−127.852	−0.31	−3.6	0.478	−126.565	0.71
	9	−3.5	−0.130	−128.561	−0.86	−1.6	0.478	−126.279	0.94
σ	T	$\lambda\times 100$	$r\times 100$	W	$\Delta WC(\%)$	$\lambda\times 100$	$r\times 100$	W	$\Delta WC(\%)$
(b) *Lower T with exogenous BC, robustness*									
1.10	52	0.000	0.723	−1125.880		0.000	0.723	−1125.880	
	46	0.362	0.953	−1126.027	−0.13	0.280	0.723	−1126.802	−0.81
	40	0.720	1.195	−1126.312	−0.38	0.500	0.723	−1127.118	−1.09
2.00	21	0.000	0.478	−127.461		0.000	0.478	−127.461	
	15	0.118	0.556	−127.559	−0.77	0.110	0.478	−127.855	−0.31
	9	0.267	0.668	−127.667	−0.16	0.237	0.478	−128.194	−0.57

of individuals over assets, and this impact can be quantitatively substantial. Do these conclusions rest on the specific choices of parameters of the baseline case? Similar experiments have been conducted on the alternative parametric examples presented in Table 4 which differ from the baseline in the values of σ and T. It turns out that the conclusions from the baseline model carry over to all these cases as well. To supply some specific examples, Table 6(a) displays outcomes for the settings in rows 4 and 7 of Table 4.

4.1.2. *Exogenous borrowing constraint*

In the second type of experiment, the borrowing constraint is instead held fixed at its benchmark value. Under these conditions, the default rate λ may change as an endogenous variable. This exercise thus resembles in its assumptions the analysis in Athreya (2002) and Li and Sarte (2002). Like in those papers, and consistently with the U.S. code, it will be assumed that only individuals with income below the median can file for bankruptcy. In the model, this condition amounts to exogenously imposing that $b(y_1) = \underline{b}$. Section A of Table 7 displays the consequences of reducing T by the same amount as in the previous experiment. As T is reduced, the borrowing interest rate r and the default risk λ both increase, whereas the lending interest rate, $r-\lambda$, declines. The increase in default is characterized by a shift in the bankruptcy threshold for the low-income individuals, $b(y_2)$, away from the borrowing constraint.[20] It leads to annual bankruptcy rates of about 0.2% and 0.5%, and a

[20]The restriction on $b(y_1)$ is not binding with $T = 26$ but it is with $T = 20$.

Table 7
Lower T with exogenous BC, baseline model

T	$\underline{b}$	$\lambda \times 100$	$r \times 100$	$b(y_2)$	j_2	W	$\Delta WC(\%)$
A. General equilibrium							
32	−5.20	0.000	0.635	−5.200	1	−237.569	
26	−5.20	0.220	0.773	−5.169	9	−237.771	−0.17
20	−5.20	0.464	0.940	−5.111	14	−237.982	−0.35
B. Small open economy							
32	−5.20	0.00	0.635	−5.200	1	−237.569	
26	−5.20	0.184	0.635	−5.176	8	−238.312	−0.62
20	−5.20	0.381	0.635	−5.138	12	−238.729	−0.97

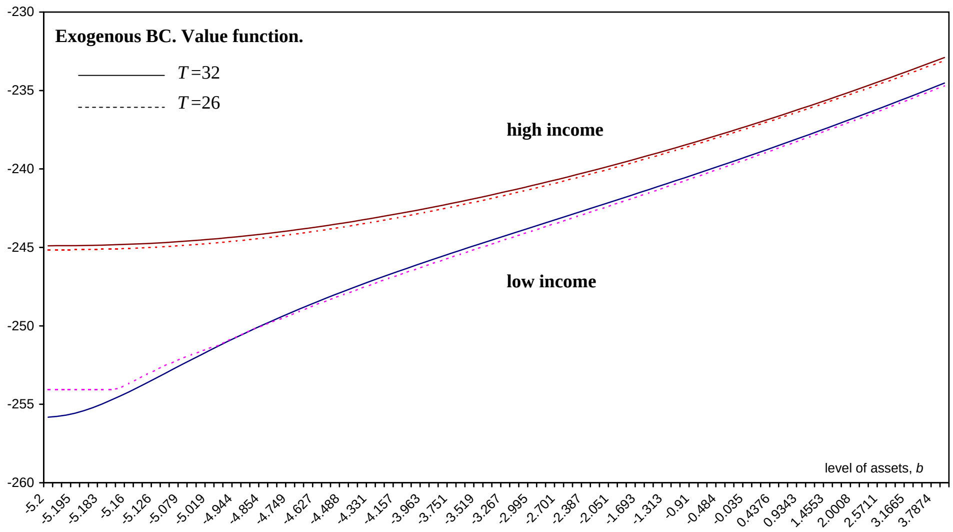

Fig. 7. Exogenous BC. The effect of lower T on the value function.

mass of individuals restricted from trade (i.e., with $z = 1, \ldots, T$) equivalent to 0.7% and 1.14% of the total. The rise in the equilibrium interest rate precisely counters the surge in the demand for credit at levels in or near the bankruptcy region. Turning to welfare, when T is lowered the measure W displays a reduction which, as indicated by ΔWC, is in the order of 0.2% and 0.4% in equivalent consumption units.

To understand the factors at work, one needs to look again at the fine details of the implications for utility levels and the distribution. Note that when, unlike in the benchmark equilibrium, there is positive default, the fraction of the population which is restricted from trade must be accounted for although their quantitative influence on aggregate measures will be necessarily minor. Once again the discussion centers around the change in T from the baseline value of 32 to the lower 26. Fig. 7

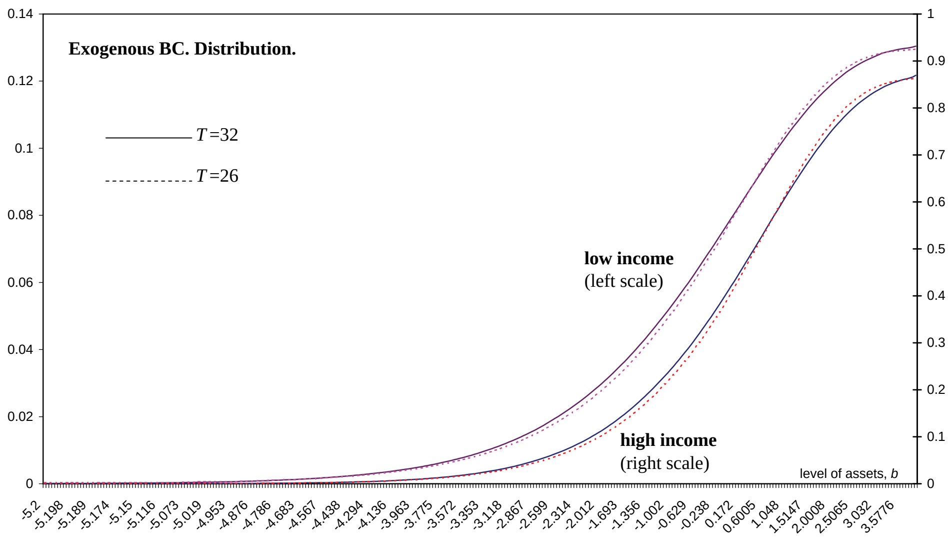

Fig. 8. Exogenous BC. The effect of lower T on the wealth distribution.

displays the value function. Among those individuals that participate in the economy (i.e., $z = 0$) utility falls relative to the benchmark equilibrium, except for low-income individuals with high levels of debt on or near the default region. However, the mass of agents who experience a gain in utility is very small. On their part, the group of individuals that now become restricted from the markets (i.e., $z > 0$) enjoy an average level of utility -243.08 which is well below the average. Fig. 8 depicts the cumulative distribution function over asset levels. The distribution shifts its mass towards the median and bottom region of assets, and does not have an obvious impact on computed welfare. Thus the negative response of welfare is mainly caused by the direct changes in utility levels.

These shifts in the value function and the wealth distribution are ultimately caused by the adjustment of three variables: the interest rate, the risk spread, and the low-income individual's default rule $b(y_2)$. Fig. 9 helps disentangle their separate roles by drawing the value function when only either the interest rate and the spread or the default behavior change, provided the rest of variables remain as in the baseline equilibrium. The increase in bankruptcy increases utility levels for all non-bankrupt individuals, but this is only clearly visible for high-debt low-income states. These gains arise because individuals smooth consumption further by borrowing more heavily in bad states given that discharging debts through bankruptcy has become cheaper. On the other hand, the joint adjustment of the interest rate and the spread have a predominantly negative effect. Therefore, the reduction in welfare following a lower T reflects the adverse increase in the interest rate and the risk spread. Although increased default does certainly have a positive risk-sharing impact on utility levels, it is quantitatively too weak to manifest itself in equilibrium.

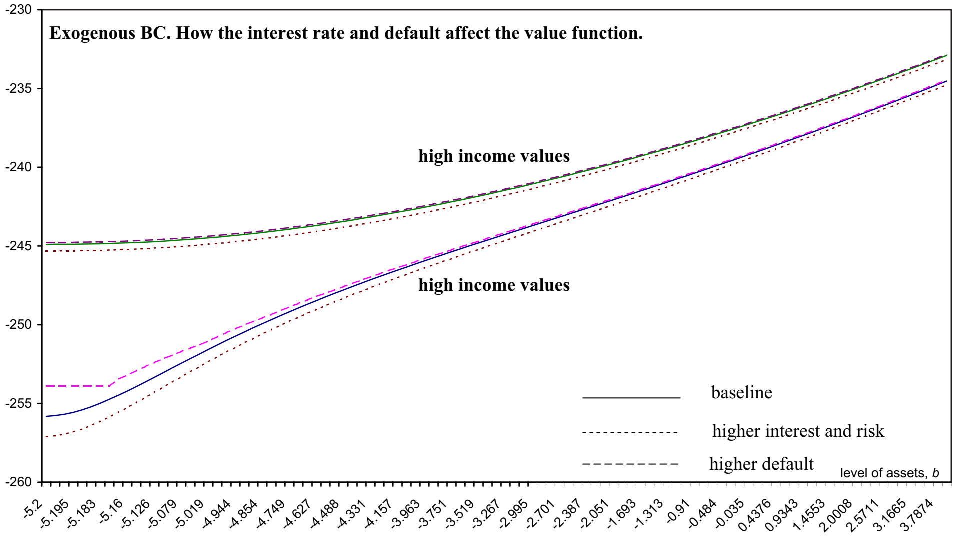

Fig. 9. Exogenous BC. The effect of r and λ, and default $b(y_2)$ on the value function. Higher interest and risk: $r = 0.00773$ and $\lambda = 0.0022$; higher default: $b(y_2) = -5.169$.

In this experiment with an exogenous BC, just as in the one presented before, there is little room for changes in the wealth distribution. To study such changes, the analysis turns now to the case of the small open economy where the interest rate is given. Section B of Table 7 shows the results from lowering T in this case. Default increases but less than in the closed economy because the interest rate is held at a lower level. The measure of welfare experiences a sharper decline than in the closed economy which, as shown by ΔWC, is in the order of 0.6% and 1.0% in consumption units. The reaction of the value function and the wealth distribution will explain this. Figs. 10 and 11 display them for the baseline equilibrium and the case with the lower $T = 26$. The value function experiences a modest upward shift at high-debt low-income states due to the broader risk-sharing opportunities. The distribution, on the other hand, shifts towards the left so that the mass of individuals in low-utility states rises. This higher concentration at low levels of assets has a stronger effect on W than the rise in utility levels and dictates the recorded fall in aggregate welfare.

To sum up, with an exogenous BC, the positive risk-sharing implications of rising default rates play a small part in the welfare consequences of easing the default penalty. In general equilibrium, welfare declines because of the rise in the lending interest rate and the risk spread, but the total impact is quantitatively modest. In a small open economy, welfare declines because of the adverse shift in the wealth distribution, and this impact can be quantitatively substantial. These qualitative and quantitative outcomes stand up to changes in parametric assumptions. Table 6(b) reports similar exercises for the settings presented in rows 4 and 7 of Table 4.

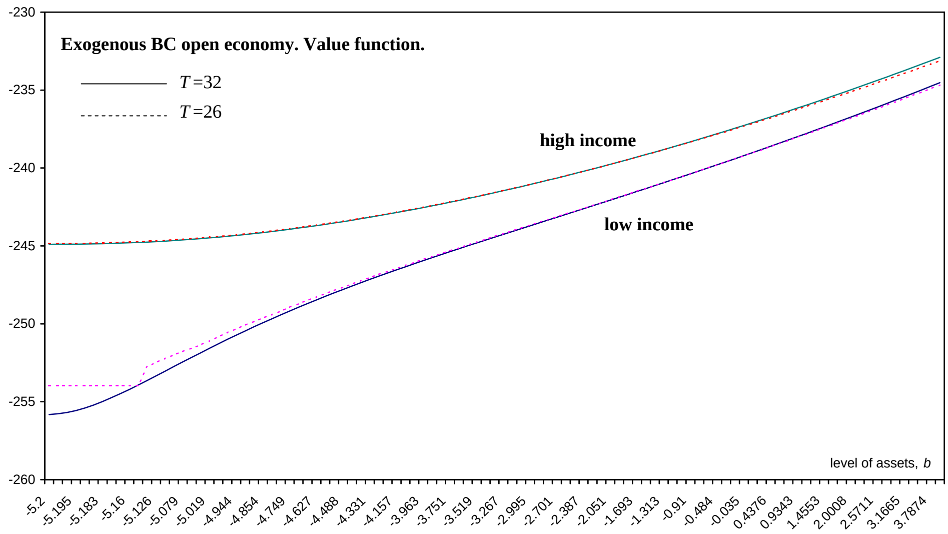

Fig. 10. Exogenous BC in the small open economy. The effect of lower T on the value function.

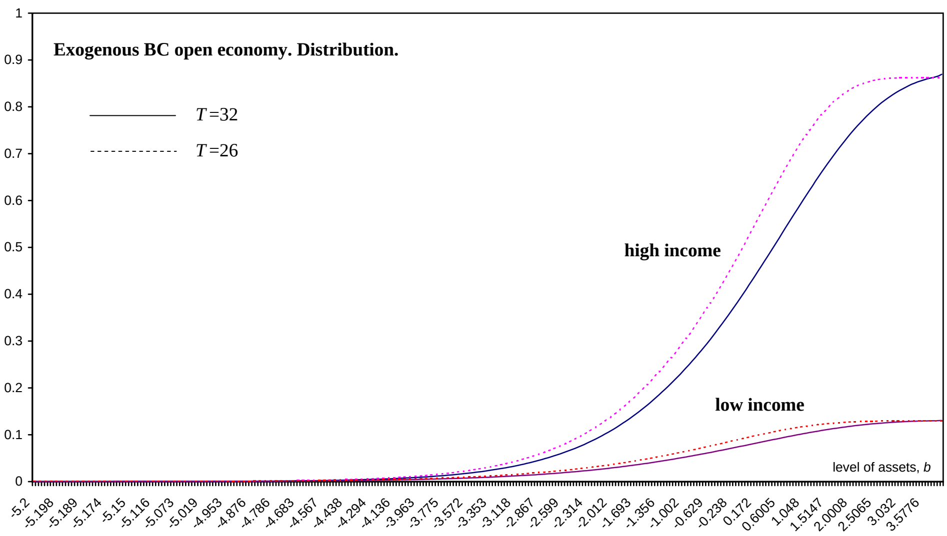

Fig. 11. Exogenous BC in the small open economy. The effect of lower T on the wealth distribution.

4.2. Social insurance

The exercise with income dispersion consists of a mean-preserving reduction in the spread between y_1 and y_2, which can be related to a change in social insurance policy

Table 8
Lower income variability with endogenous BC, baseline model

y_1	y_2	$\underline{b}$	$\lambda \times 100$	$r \times 100$	$b(y_2)$	j_2	W	$\Delta WC(\%)$
A. General equilibrium								
1.000	0.100	−5.20	0.000	0.635	−5.200	1	−237.569	
0.990	0.167	−3.80	0.000	0.530	−3.800	1	−237.621	−0.04
0.980	0.233	−2.70	0.000	0.380	−2.700	1	−237.714	−0.12
B. Small open economy								
1.000	0.100	−5.20	0.000	0.635	−5.200	1	−237.569	
0.990	0.167	−3.70	0.000	0.635	−3.700	1	−236.896	0.85
0.980	0.233	−2.50	0.000	0.635	−2.500	1	−236.387	1.00

deployed through taxes and transfers across individuals in the two income groups. In this crude formulation, every individual participates in this scheme irrespective of her bankruptcy status.

4.2.1. Endogenous borrowing constraint

As before, in the first type of experiments the borrowing limit is an endogenous variable. Section A of Table 8 reports the consequences of closing the baseline income differential by about 8% and 17%. The figures on the first row correspond to the benchmark equilibrium. The reduction of individual risk variability makes individuals more inclined to default since the temporal exclusion from financial markets now becomes less costly in terms of risk sharing. The response of the banks is to tighten credit conditions to prevent that from happening, hence the increase in $\underline{b}$. Two forces with opposite sign influence the interest rate: the more restrictive credit opportunities push the interest rate down, however more social insurance drives the market clearing interest rate up as the demand for (precautionary) saving declines. The fall in the interest rate shows that the effect of the stricter credit limit prevails. The welfare consequences are negative as indicated by the modest reduction W in the order 0.04% and 0.1% in equivalent consumption units.

To understand these welfare implications, one must study the associated changes in the cumulative wealth distribution and value functions by income levels, respectively. The analysis focuses on the change between the baseline and the lower income variability given by $(y_1, y_2) = (0.99, 0.167)$. In Fig. 12 the distribution shifts in the 'neutral' manner which is typical of a market-clearing equilibrium. So most of the action must come from the direct impact on utility levels. Fig. 13 shows that utility levels, if anything, tend to decline, particularly at high levels of debt. The explanation must be found in the combined effect of the smaller income gap, the lower interest rate, and the tighter BC. Fig. 14 breaks down their separate impact of the value function. The lower income variability increase utility throughout. The lower interest rate also has a positive yet modest effect on utility levels. The stricter BC shifts the value function sharply downwards. As seen before, more stringent

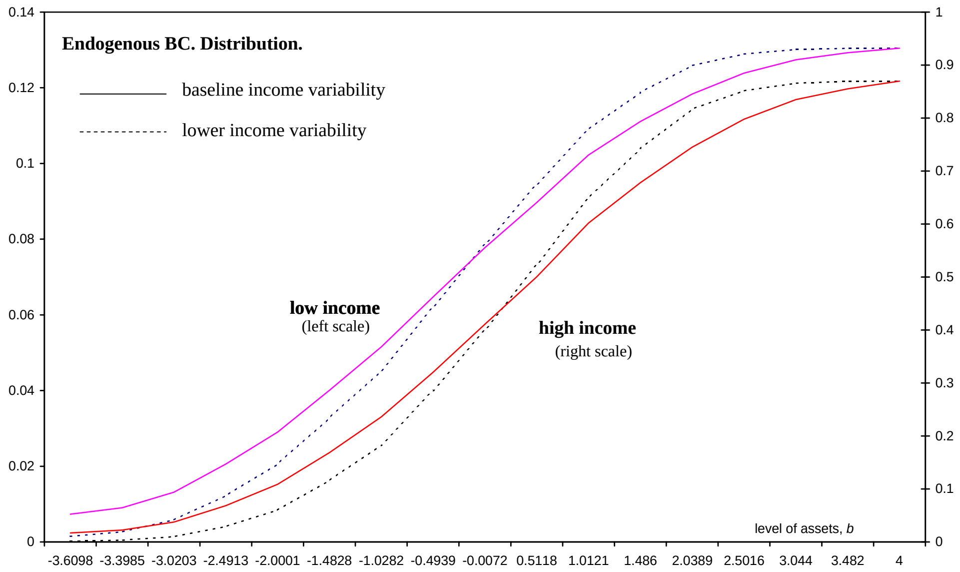

Fig. 12. Endogenous BC. The effect of lower income variability on the wealth distribution. Baseline income variability: $(y_1, y_2) = (1.00, 0.100)$; lower income variability: $(y_1, y_2) = (0.99, 0.167)$.

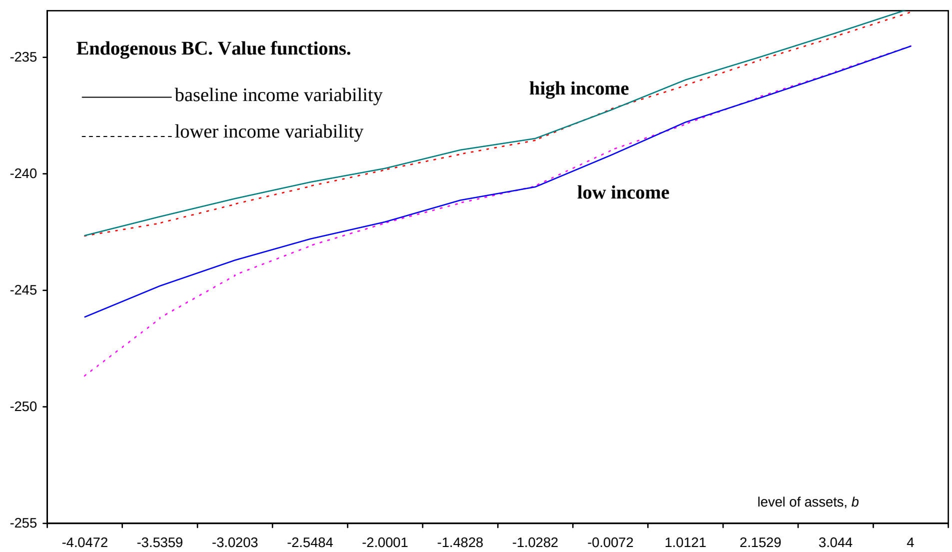

Fig. 13. Endogenous BC. The effect of lower income variability on the value function. Baseline income variability: $(y_1, y_2) = (1.00, 0.100)$; lower income variability: $(y_1, y_2) = (0.99, 0.167)$.

credit constraint have an adverse impact on the opportunities for risk sharing which outweighs the positive influence of the lower interest rate and, in this case, even the direct social insurance impact of a lower income variability.

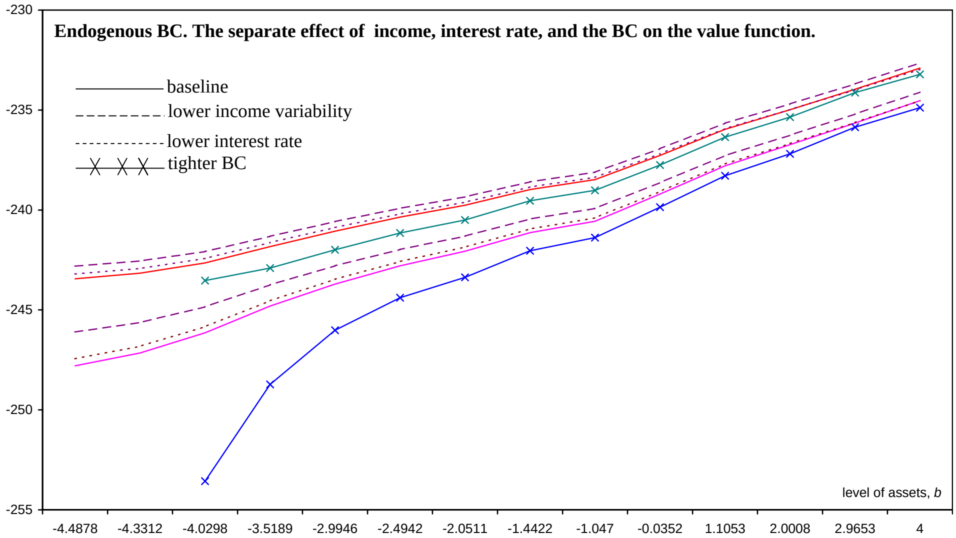

Fig. 14. Endogenous BC. The effects of income variability, interest rate, and BC on the value function. Lower income variability: $(y_1, y_2) = (0.99, 0.167)$; lower interest rate: $r = 0.053$; tighter borrowing constraint: $\underline{b} = -3.80$.

The implications of a narrower variation in individual income in the small open economy are reported in Section B of Table 8. The borrowing constraint becomes tighter for reasons already discussed earlier. Welfare W improves by about 0.8% and 1.0% in equivalent consumption units. It is true that the more restrictive credit conditions still have a negative risk-sharing impact on utility levels similar to that represented in Fig. 13. However, since the return to saving does not fall in the open economy, the tighter BC also leads to a shift in the wealth distribution comparable to that shown in Fig. 6 earlier on.

Summing up, in a closed economy with an endogenous BC, lower individual income variability leads to more restrictive credit conditions which cancel out any direct gains which might arise from social insurance or a lower interest. The net impact is negative but quantitatively small. For a small open economy, however, the more restricted credit conditions push the mass of individuals towards higher wealth positions and dictates an aggregate welfare gain which can be substantial. These results do not depend on the choice of baseline parameters. The same exogenous changes in income variation have been studied in the settings presented in Table 4 and the same conclusions carry over.

4.2.2. *Exogenous borrowing constraint*

In the second experiment the borrowing limit is instead fixed and the rate of default becomes endogenous. As before in Section 4.1.2, the constraint $b(y_1) = \underline{b}$ is imposed. It will also be assumed that bankruptcy can only be declared if debt exceeds

Table 9
Lower income variability with exogenous BC, baseline model

y_1	y_2	$\underline{b}$	$\lambda \times 100$	$r \times 100$	$b(y_2)$	j_2	W	$\Delta WC(\%)$
A. General equilibrium								
1.00	0.100	−5.20	0.000	0.635	−5.200	1	−237.569	
0.99	0.167	−5.20	0.800	1.250	−5.060	17	−237.994	−0.36
0.98	0.233	−5.20	11.000	9.950	−4.030	42	−242.916	−4.35
B. Small open economy								
1.00	0.100	−5.20	0.00	0.635	−5.200	1	−237.569	
0.99	0.167	−5.20	0.575	0.635	−5.138	12	−238.936	−1.14
0.98	0.233	−5.20	2.700	0.635	−4.995	20	−238.901	−1.11

75% of average annual income, or $b(y_2) \leqslant -4.0$.[21] Section A of Table 9 displays the response of the endogenous variables to mean-preserving reductions in the spread of income realizations. The borrowing interest rate r and the default risk both increase, whereas the lending rate declines. The increase in default follows from the shift in the bankruptcy threshold for low-income individuals away from the borrowing limit. It leads to annual bankruptcy rates of about 0.8% and a 11%, with the mass of individuals restricted from trade equivalent to 2.8% and 21.9% of the total population. Turning to welfare, when income variability is reduced the measure W experiences a decline which is in the order of 0.4% and 4% in equivalent consumption units.

The changes in the distribution and the value function between the baseline equilibrium and the one with $(y_1, y_2) = (0.99, 0.167)$ can be examined in Figs. 15 and 16. Although there are substantial direct utility gains at low-income states, the utility drop at high-income states drives the negative response of overall welfare. To identify the role played by the various key variables, Fig. 17 draws the value function corresponding to four different situations: the baseline equilibrium; when only the income gap changes; when only the interest rate and the risk-spread change; and, finally, when only the income gap and the default decisions change. The joint effect of the interest rate and the risk spread is negative. On the other hand there is a positive effect for the narrower income gap which is reinforced by the rise in default as a means to improve risk-sharing. In the end, the negative interest-rate factor prevails.[22] Note this effect is far more dramatic for the example in Table 9 with the lowest income variability $(y_1, y_2) = (0.98, 0.233)$.[23]

For the small open economy, Section B of Table 9 shows the consequences of a narrower income gap. Default increases but by less than in a closed economy. The measure of welfare shows a sharper decline than in the closed economy, in the order

[21]This restriction guarantees existence of an equilibrium when default rates become very large, and is only binding in those situations.

[22]Comparison of the curves for low-income in Figs. 16 and 17 suggest that the negative interest-rate effect will be much weaker when the smaller income gap and the higher default are already accounted for.

[23]This is the case where the constraint $b(y_2) \leqslant -4$ binds.

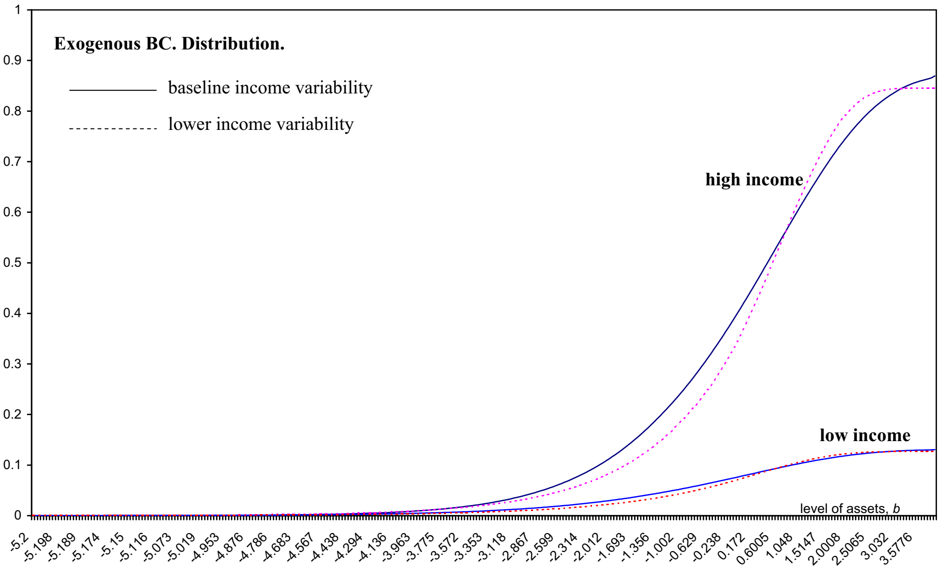

Fig. 15. Exogenous BC. The effect of lower income variability on the wealth distribution. Baseline income variability: $(y_1, y_2) = (1.00, 0.100)$; lower income variability: $(y_1, y_2) = (0.99, 0.167)$.

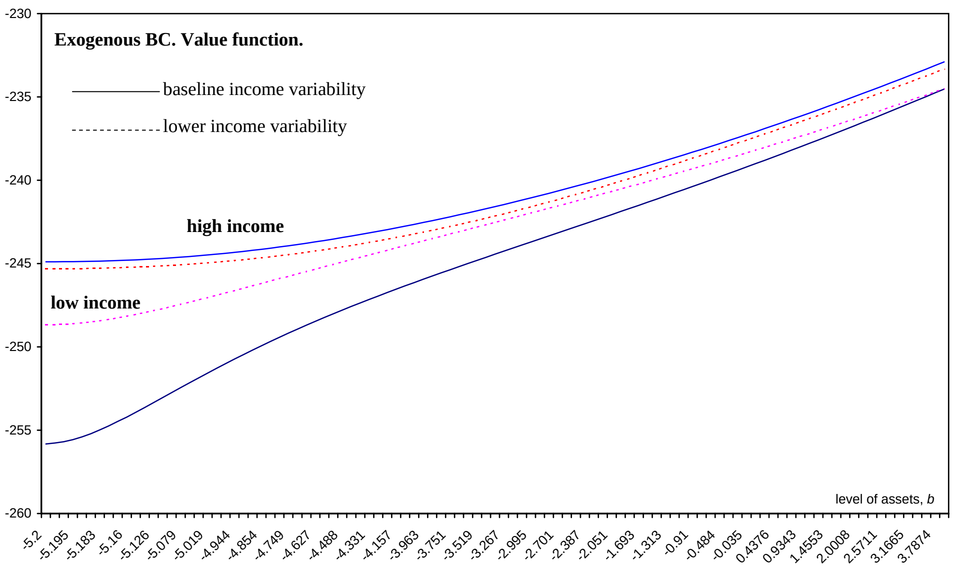

Fig. 16. Exogenous BC. The effect of lower income variability on the value function. Baseline income variability: $(y_1, y_2) = (1.00, 0.100)$; lower income variability: $(y_1, y_2) = (0.99, 0.167)$.

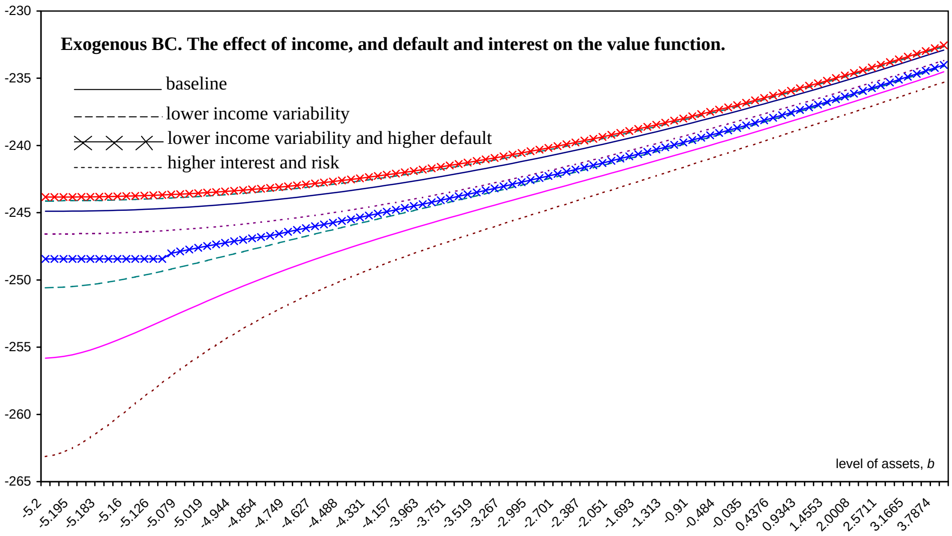

Fig. 17. Exogenous BC. The effects of income variability, the interest rate, and default on the value function. Lower income variability: $(y_1, y_2) = (0.99, 0.167)$; lower income variability and higher default: $(y_1, y_2) = (0.99, 0.167)$ and $b(y_2) = -5.060$; higher interest rate and risk: $r = 0.0125$ and $\lambda = 0.008$.

of 1% in equivalent consumption units. The dominant factor is a shift to the left of the wealth distribution which is comparable to that in Fig. 11. As the interest rate does not increase, individuals are inclined to borrow more. This effect outweighs the combined positive influence of lower income variability and higher default on individual utility levels.

In sum, with an exogenous BC the positive risk-sharing implications of rising default rates and a smoother income play a relatively small part in the welfare consequences of a reduction in income variability. In the closed economy, welfare declines because of the rise in the lending interest rate and spread, but the total impact is quantitatively modest. In a small open economy, welfare declines because of the adverse shift in the wealth distribution, and this impact can be quantitatively substantial. Similar exercises have been run for the settings presented in Table 4 which show that these qualitative and quantitative outcomes stand up to changes in parametric assumptions.

5. Conclusion

This paper introduces the endogenous determination of the borrowing constraint and default risk in a typical model of an exchange economy with incomplete markets, idiosyncratic risk, and a continuum of agents. The same setup can be used to accommodate situations where the borrowing constraint is instead fixed exogenously. The model is used to study quantitatively the welfare and positive consequences of, first, a reduction in the period of exclusion from markets which penalizes bankruptcy and, second, social insurance. Formulating and solving this model, and assessing quantitatively the role of various interacting mechanisms for welfare are two notable contributions of this paper to an emerging literature.

In general equilibrium, a lower cost to declaring bankruptcy tends to reduce welfare regardless of whether the borrowing limit is endogenous or exogenous. The type of mechanisms driving the result are, however, very different in each case. When the borrowing limit is endogenous, it becomes tighter and its adverse risk-sharing effect wipes out the gains from a lower interest rate. When the borrowing limit is exogenous, the rise in the interest rate and the risk premium outweigh the risk-sharing gains associated with a higher level of default. Quantitatively, a reduction in the default punishment period by 1 and 2 years leads to a welfare loss in the range of 0.2% and 0.5% in equivalent consumption units. In contrast, in the special case of a small open economy the welfare consequences of a softer bankruptcy penalty are driven by the ensuing change in the wealth distribution. If the borrowing limit is endogenous, credit is restricted, individuals become more concentrated at higher levels of wealth, and welfare increases in the order of 0.7% and 1.2%. If the borrowing limit is exogenous, a higher default risk is incurred, individuals become more concentrated at lower levels of wealth, and welfare decreases by about 0.7% and 1.0% in consumption units. The conclusion in a small open economy depends thus a great deal on whether the borrowing constraint is endogenous, both qualitatively and quantitatively. On the other hand, the introduction of social insurance has very similar implications, including the invariably modest welfare losses in the closed economy, and the importance of the endogeneity of the borrowing constraint for the welfare gains in a small open economy.

These conclusions are based on a very stylized model of the economy and financial institutions though. First, with an endogenous borrowing constraint there is no default. Although the case of positive default under a given borrowing constraint has also been considered and may be informative, a theory where both the borrowing constraint and the default rate may respond to changes is a natural next step. Second, whereas welfare effects may not depend dramatically on the endogeneity of the borrowing constraint, the interest rate does. A model of a production economy with capital would therefore be a necessary extension. Third, this paper does not account for the finer details of actual bankruptcy laws, such as the possibility of saving after declaring bankruptcy, or the existence of exemptions. Dealing with these aspects would help assess more firmly the practical implications of bankruptcy regulations.

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Appendix A. Details on computation

A.1. Discretization

The state space S is discretized to calculate $\Phi(s)$ as derived from the optimal decision rules $b'(s)$ and $d(s)$ (or S_0) and the transition probabilities $\pi(y' \mid y)$. I let $b \in \{b_1, \ldots, b_j, \ldots, b_{j\max}\}$ and define

$$S_0 = \{(b_{j(y)}, y, 0), \ldots, (b_{j\max}, y, 0) | y \in \{y_1, y_2\}\},$$

where $b_{j(y)}$ is the smaller value on the grid that is no smaller than the default threshold $b(y)$ defined above. The case with zero default $S_0 = S$ is equivalent to $j(y_1) = j(y_2) = 1$. The grid for bonds has 101 unevenly spaced elements with an upper bound $\overline{b} = 4$ (there are 303 points in the finer grid used to approximate the stationary distribution). The points on the grid b_k for $k = 1, \ldots, N$ are determined as $b_1 = \underline{b}$, and for $k > 1$,

$$b_k = \underline{b} + (\overline{b} - \underline{b}) \frac{k^{2.35}}{N^{2.35}}.$$

A.2. Distribution

On this discreet setup, the law of motion for the distribution is given by

$$\Phi'(s') = \sum_s Q(s, s')\Phi(s),$$

where $Q(.,.)$ is the transition function. The natural way to compute the stationary distribution is to iterate this equation until convergence. It is useful to distinguish different cases:

- If $z' = 0$:

$$\Phi'(b', y', 0) = \begin{cases} \sum_y \pi(y' \mid y)\Phi(b^{-1}(b', y), y, 0) & b' < 0, \\ \sum_y \pi(y' \mid y)\Phi(b^{-1}(b', y), y, 0) + \sum_y \pi(y' \mid y)\Phi(0, y, 1) & b' \geqslant 0. \end{cases}$$

  The $b^{-1}(b', y)$ is the value bonds such that, given income y, leads to a choice b'. If $b' > b'(b_{j\max}, y, 0)$ then $\Phi(b^{-1}(b', y), y, 0) = \Phi(b_{\max}, y, 0)$; on the other hand, if $b' < b'(b_{j(y)}, y, 0)$ then $\Phi(b^{-1}(b', y), y, 0) = 0$.
- If $z' \in \{T - 1, T - 2, \ldots, 2, 1\}$:

$$\Phi'(b', y', z') = \begin{cases} 0 & b' < 0, \\ \sum_y \pi(y' \mid y)\Phi(0, y, z' + 1) & b' \geqslant 0. \end{cases}$$

- If $z' = T$:

$$\Phi'(b', y', T) = \begin{cases} 0 & b' < 0, \\ \sum_y \pi(y' \mid y)\Phi(b_{j(y)}, y, 0) & b' \geqslant 0. \end{cases}$$

In a stationary distribution, the mass of agents that participate and that are excluded will be constant. In the computations this condition is imposed in each step by normalizing the mass of agents within the economy to unity. This device improves the convergence properties of the procedure.

A.3. Default

The optimal defaulting decisions are given by a pair $(b(y_1), b(y_2))$. To calculate this, one has to search over pairs $(b(y_1), b(y_2))$ (i.e., in the computation $(j(y_1), j(y_2))$). One potential difficulty is that both values affect the participation condition of the two income levels at the same time. The procedure adopted here consists of searching for a pair where the two participation values become non-negative. To speed things along, the region of search is first narrowed as follows. Take an initial $b(y_1) = \underline{b}$. Then check the two participation values at $b(y_2) = \underline{b}$. If both are positive this is the pair sought, otherwise evaluate the participation value in the low-income state $PART_2, y_2$, at $b(y_2) = 0_-$. If it is negative then fix a new larger $b(y_1)$ and evaluate the participation values at $b(y_2) = \underline{b}$ and start again. Otherwise, search for the first $b(y_2) \in [\underline{b}, 0_-]$ such that participation in the low-income state becomes positive. Check it is also positive for the high-income state, otherwise fix a new larger $b(y_1)$ and evaluate the participation values at $b(y_2) = \underline{b}$ and start again. Note these calculations are done on the coarse grid.

The possibility of multiple S_0 is handled as follows. Use the above iterative procedure starting with $b(y_1) = \underline{b}$. If it leads to $b(y_1) > b(y_2)$ the pair found characterizes the unique S_0. If it leads to $b(y_1) \leqslant b(y_2)$, this pair may or may not be the unique S_0. To check existence, use the iterative procedure of the previous paragraph but imposing $b(y_1) \geqslant b(y_2)$. Such a type of equilibrium does not exist if either it does not reach an end or it delivers $b(y_1) = b(y_2)$.

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